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A TREATISE ON ASTRONOMY.

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A

TREATISE ON ASTRONOMY,

FOR THE

USE OF COLLEGES AND SCHOOLS.

BY

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PREFACE.

Astronomy is a science admitting of so many methods of treatment, that a few words will be useful to explain the scope and object of the present work.

It is essentially a student's book. Prepared originally for the use of the Author's pupils in the University, it embraces all those branches of the subject which have, from time to time, been recommended by the Board of Mathematical Studies; but the easier, and by far the larger, portion, adapted to the first three days of the Examination for Honours, may be read by the more advanced pupils in many of our schools.

It is difficult to be original in an elementary work, and on a subject which has occupied men's thoughts from the earliest times. The Author's aim has been rather to convey clear and distinct ideas than to affect originality; and it is hoped that where he has deviated from the beaten track it has been at no sacrifice of simplicity.

Before the invention of clocks the work of the practical astronomer was of a most wearisome nature. Every observation to determine the

position of a celestial body would require the measurement of its distances from other bodies, whose positions were known; and this, besides being liable to a variety of instrumental and other errors, would involve long and tedious calculations. The perfection to which clocks have been brought has so greatly simplified this, by enabling the observer to make use of the uniform rotation of the earth as an element of observation, that the clock is almost as important an instrument in the observatory as the telescope. It was, therefore, thought that a short description of its construction, and of the principles on which its accuracy depends, would not be out of place in a Treatise on Astronomy.

In this second edition the whole work has been very carefully revised and numerous alterations and additions have been made. Some are merely verbal, others more important; but the order of the chapters and the general arrangement have been preserved.

The Author desires to express his thanks for the valuable suggestions for which he is indebted to the kindness of several friends.

CAMBRIDGE, *May* 1, 1874.

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For an elementary course, the student may confine his attention to the following articles:

1—35, 40—66, 71—73, 83—147, 158, 159, 161, 163, 165, 166, 170, 171, 176—184, 189, 190—196, 199—210, 224, 225, 234, 235, 240, 241, 246—248, 250, 254—256, 258, 262—264, 268—270, 271—276, 283—284, 286—289, 291, 292, 293—309, 311—337, 348—351, 352—359, 361—364, 366—368, 371—385.

ASTRONOMY.

CHAPTER I.

THE STARS.

1. SUPPOSE an observer, on a clear winter evening, to station himself on some elevation whence he may have an unobstructed view of the heavens. He will see a large number of stars, of different degrees of brilliancy, scattered without apparent law over the celestial vault. He will imagine the shape of the vault to be spherical, and although the idea he may form of the size of this sphere will probably be vague and even different at different times, yet the parts over head will not seem so far off as those near the horizon, the effect being the same as if the visible portion were a segment of a sphere less than a hemisphere.*

2. After some time the observer will notice that a change has taken place; some of the stars have disappeared below the horizon on one side, and new ones have arisen in the

* That this spherical boundary has no real existence we shall have full proof of when we find that the sun, the moon, the planets, the stars, &c., which, to us, all seem situated in the surface are at immensely different distances.

The blue vault is in fact nothing but an imaginary background formed by the atmosphere that surrounds us; and the appearance of greater distance in the parts near the horizon is an optical illusion, due probably to the fact that the greater thickness of atmosphere through which the bodies there are seen absorbs more of their light, and gives them an amount of indistinctness, such as we know to be the effect of greater distance in the case of those objects which are accessible to us.

opposite quarter. But he will remark that there is no alteration in their relative positions; and if, by means of a sextant, or any other instrument for measuring angular distances, he determines the angle between any two stars, he will find it to remain constant, and this, not only throughout that night, but after any interval of months or years.* The motion is, therefore, a general one of the whole system of stars, and in consequence of their fixed relative positions they are called *fixed stars*, to distinguish them from the planets, or wandering stars, a small number of bodies, whose positions among the others are continually changing.†

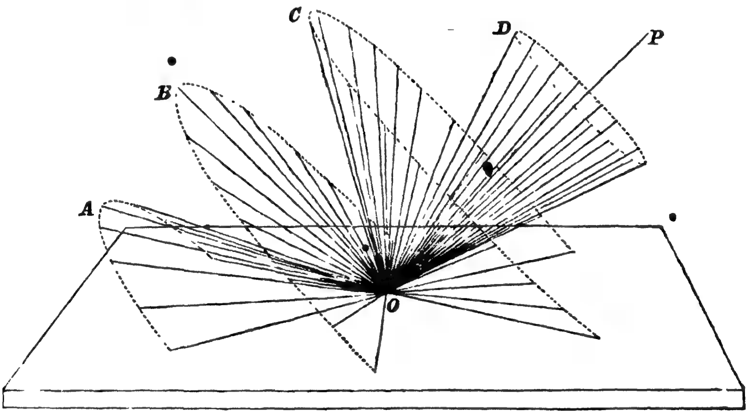
3. To determine roughly the nature of this general motion, let the observer be furnished with a number of threads or wires of equal lengths, and having a common fixed extremity, and let one of these threads be directed towards any arbitrarily chosen fixed star and there made fast: after any interval of time let a second thread be directed to the same star, a third after another equal interval, and so on. It will be found that the ends of the

* Another curious illusion may here be noticed. Any one watching the night sky could not fail to perceive that the distance between two neighbouring stars appears to become less as they rise higher; and the same illusion, but much more striking, is observed in the case of the full moon, which seems very much larger when rising than when high up in the heavens. This illusion is only another form or rather a consequence of that mentioned in the previous note. We see the moon attached, as it were, to the spherical canopy that surrounds us; and, consequently, when near the horizon occupying that part which appears furthest from us, then coming nearer and nearer as it ascends. Supposing, therefore, that its actual magnitude does not alter, we should look for a corresponding increase in the angle which it subtends, but, contrary to expectation, this angle remains almost unchanged; and we find it difficult to resist the impression that the full moon near the horizon is a much larger body than when high up. The same explanation applies of course to the apparent change of distance between neighbouring stars.

† Some of the stars have been ascertained to have a slow "proper motion," but to detect this, observations of the most delicate kind are required, extending over a long interval of time; and there is every reason to believe that, as our methods of observation still further improve, it will be found that no star can be strictly called fixed. (Chap. XVI.)

threads will all be situated in the circumference of a circle, or, which is the same thing, the threads themselves will lie in the surface of a right cone with a circular base; the angle of the cone will be different for different stars, but whatever star may have been chosen, the axes of all the cones will always be found to have a common direction inclined to the horizontal plane at an invariable angle, so long as the observer does not change his place of observation.

4. Suppose *OP* to be the direction of this common axis above the plane, then those stars which are furthest from *OP* will only just appear above the horizon; others will shew but a small portion of the cone as *A*, and as we approach



OP a larger portion of each cone is gradually obtained as at *B*, *C*, and some of the stars will not set at all, but will give a cone completely above the horizon as at *D*. In the case of those stars whose direction is at right angles to the axis *OP*, the cone becomes a plane as at *B*.

These observations point to the conclusion that the stars have a motion of revolution round this axis *OP*. The return of day will, it is true, arrest the observations before the circuits have been completed; but night after night the

same stars will be found going over the very same courses, except that each night they begin a few minutes earlier than the night before; and if the observations be repeated a few months later, the stars will be then seen completing those parts of their orbits above the horizon which the day-light had previously hindered us from following.

5. Furthermore, it will be found that the consecutive threads belonging to the same star, provided they be fixed, as we have supposed, at *equal* intervals of time, are separated by equal angles, and therefore the motion of each star is uniform. The angular velocity about the axis will also be the same for all the stars; because, consistently with what has been said before of their unaltered relative position, they will all be found to accomplish their revolution in precisely the same time.

The observations here indicated are very rough, but more accurate methods will fully confirm the results, if proper allowance be made for the slight distortion of the path caused by refraction, as hereafter explained.

Visible horizon, Dip of the horizon.

6. Before proceeding further it will be useful to ascertain, at least approximately, the shape of the earth, from which the observer has to make all his observations.

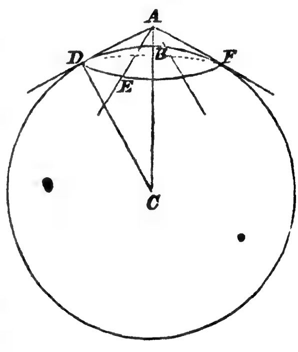
From the place he occupies he can see only a very limited portion of the surface, but he will be able to remark that the apparent boundary line where the heavens and earth seem to meet, or the *visible horizon*, as it is called, is circular, except where broken by hills or other irregularities; and if he has a calm sea for boundary, the circle will seem perfect. This appearance he will find to follow him in whatever direction, and however far he may travel: the visible horizon is circular in all places and for all heights above the sea-level.

Now a sphere is the only surface which has this property of always giving a circle as the curve of contact of a number of tangent lines drawn from *any* external point. Therefore, so far as this rough test goes, the earth is a sphere limited in extent.*

Without leaving his first station, the observer would be able to infer from the disappearance of some stars below the horizon on one side, and their re-appearance the next night on the other, having evidently passed under the earth in the interval, that the earth is isolated in space.

The fact also of its having been so often circumnavigated proves this, and proves its limited extent as well.

7. Let DFC represent the earth regarded as accurately a sphere, C its centre A, the eye of the observer, DEF the visible horizon. Let AB be the direction of a plumb line, which will be normal to the surface of still water at B, and therefore will pass through the centre C.



ABC is called a vertical line at B. A plane at right angles to this vertical line through any point of it is called the horizontal plane at that point. When drawn through B it is called *sensible*, and when through C *rational*, but the distinction will not be of any importance.

* We shall hereafter see that it is in reality an oblate spheroid, differing however very little from a sphere, the polar diameter being 7899 miles long, and the equatorial diameter 7926 miles.

To the above proof of the convex nature of the surface may be added the way in which a ship, sailing in any direction, is gradually lost sight of as it recedes from the observer: the hull first disappears, then the lower masts and sails, and finally the top-masts. Again, the shadow of the earth, as projected on the moon in a lunar eclipse, is that which would be produced by a globular body.

8. The angle which AD makes with the horizontal plane through A is called the depression or *dip of the horizon.* It varies with the height of AB and is the complement of the angle DAC, and therefore equal to the angle at the centre DCA.

If $AB=h$, $BC=r$, $ACD=\theta$, and the arc $BD=x$, we find

$$\cos\theta=\frac{r}{r+h}, \quad x=r\theta,$$

whence the dip and the distance of the visible horizon will be known when the height above the surface and the radius of the earth are known.*

If h is small, then approximately

$$\theta=\sqrt{\left(\frac{2h}{r}\right)} \text{ and } x=\sqrt{(2rh)}.$$

Effects of a change of station.

9. Let us now return to the observations on the stars, and suppose that the observer proceeds to some new station, and there recommences his series of operations as in Art. 3. He may perhaps discover some new portions of the heavens containing new stars, or he may lose some which he could see before, but he will find, however far he may have travelled, that all the stars previously observed retain their relative positions unaltered; that they generate cones of exactly the same magnitude as before, in precisely the same time as before; and that the only change is in the inclination of the common axis of those cones to the horizontal

* On account of refraction these results will not be exactly correct. Rays from a point beyond D will be seen and reach A in a direction above DA. The angle of depression θ will therefore be diminished, and the visible distance x increased; $\theta-\frac{1}{13}\theta$ is found to be the reduced value of the one, and $x+\frac{1}{13}x$ the increased value of the other. See Art. 235.

plane, which inclination may be either greater or less according to the direction in which he has gone.

Therefore the axis about which the stars revolve seems to pass through every place on the earth's surface, and the stars preserve the same position relatively to it and to one another for every observer.

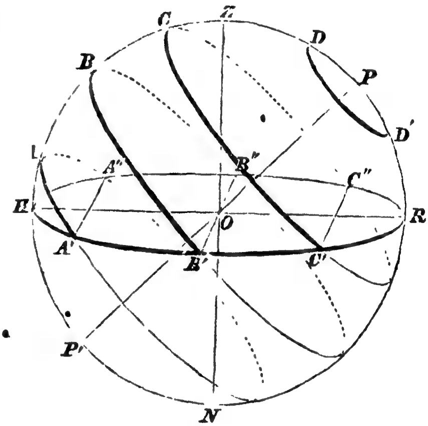
10. But this forces upon us another conclusion. If the angular distances between the stars do not alter, however much we may change our position on the earth, it must be that the earth is a mere point, compared with the distances of the stars. For, if two stars A, A' are seen under the same angle from several points P, Q, R, S, ..., then Q, R, S, ... must all be situated in the surface generated by the revolution of the segment of the circle APA' about the chord AA'; and if a new star A'' be observed under the same conditions, the points Q, R, S, ... will be also in the surfaces similarly described on the chords AA'', $A'A''$; the same for a fourth star A''' and so on; but P is the only point common to all these surfaces, therefore it is impossible that the angles subtended by the stars should really be the same at all places. The explanation is that the distances of these places from one another are too small, compared with the actual distances of the stars, to produce changes in the angles large enough to be detected even with the most refined instruments, and we shall hereafter see that angular changes due to displacements many thousand times greater are still too minute for our observation.

The earth is then a mere speck in space as regards the distances of the fixed stars: from all points on or within its surface, lines drawn to the same fixed star at the same instant will be parallel, and all the axes about which the heavens seem to revolve will remain fixed and parallel to one another.

Celestial sphere.

11. Let the observer now surround himself with an imaginary sphere, having a radius perfectly arbitrary—whether a hundred yards, or a hundred times the distance of the furthest star, is immaterial, but perhaps his conceptions will be easier with the smaller radius—this will be his *celestial sphere* which we must suppose to accompany him in all changes of position in such a manner that he may always occupy the centre. To this sphere he will refer the places of the heavenly bodies at any instant, by the points where the lines joining them with the centre cross the surface.*

Thus if *HZRN* represent his celestial sphere at the station of his first observations, the centre *O* coinciding with the vertex of the cones in fig., p. 3, then the paths of the stars which lie respectively in the surfaces of the cones *A*, *B*, *C*, *D* will be represented by the intersections of these cones with the surface of the celestial sphere, that is, by the circles *A'AA"*, *B'BB"*,



* It has been generally customary to define the celestial sphere as one having an infinitely large radius. The conclusions obtained will be just the same, but when finite magnitudes and this infinite space have both to be represented in the figure the reasoning may seem somewhat obscure.

It is only for the convenience of substituting spherical triangles for solid angles that this sphere is at all needed. There is in reality no bounding surface in the sky, and therefore this sphere does not represent anything, not even the apparent vault of the heavens, for that seems nearer to us in the portion near the zenith than near the horizon; whereas our sphere has the eye for its centre. It will be found that all problems of Astronomy, in which the Celestial sphere is used, concern only the plane and the dihedral angles which the objects subtend at the eye.

&c. The axis of rotation OP, the common axis of all the cones, will be perpendicular to the planes of these circles; these planes are therefore all parallel to one another.

The horizontal plane through O will give the great circle $HA'B'...R$, which is called the *horizon.* The point in the heavens vertically above the observer is called his *zenith* and the opposite point the *nadir.* They are given in the celestial sphere by the vertical line ZON.

The points P, P', where the axis meets the celestial sphere are called its poles; that one which is above the horizon in these regions being called the *north pole*, and the other the *south pole.*

The circles described by the stars are called *parallels*, that one BB' whose plane passes through the centre of the sphere taking the name of equator. The *equator* is therefore the great circle whose plane is at right angles to the axis of the celestial sphere.

12. The portions of the celestial sphere situated above the horizon will correspond to the visible part of the heavens. The stars situated in the equator $B'B\,B''$ have their paths bisected by the horizon, and are therefore as long above as below it; those whose paths are further from the elevated pole will be a shorter time above the horizon, those which are nearer to it a longer time; the duration in every case being the same fraction of the whole diurnal period that the arc of the parallel above the horizon is of the whole circumference of that parallel. Those stars whose parallels are entirely above the horizon, and which consequently never set, are called *circumpolar stars.*

The great circle $HZPR$ which passes through the zenith Z and the pole P is called the *meridian* of the observer.

The straight line HOR, in which the plane of the meridian meets the plane of the horizon, is called the *meridian line*, and the points R and H the *north* and *south* points.

The horizontal line which is perpendicular to the north and south line is called the *east* and *west* line, and its extremities the east and west points.

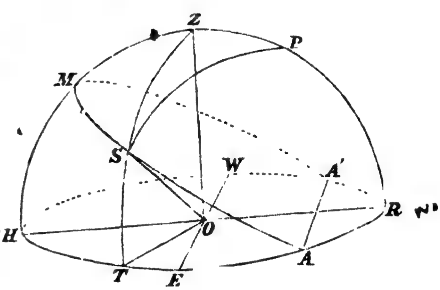
Since the meridian plane contains OP it will be perpendicular to the plane of the equator, and because it contains OZ it will also be perpendicular to the plane of the horizon, and therefore $B'B''$, the intersection of these planes, will be perpendicular to the meridian plane, and will coincide with the east and west line.

Hence *the intersections of the equator and horizon determine the east and west points.*

Altitude, Azimuth, Zenith distance, Culmination.

13. It is often necessary to describe the position which a star or other heavenly body occupies at a given instant on the observer's celestial sphere. This may be done by referring it to the meridian and horizon.

Let the figure represent the upper half of the observer's celestial sphere; Z, P the zenith and pole; HOR the meridian line; E, W the east and west points. Every plane through OZ will be a vertical plane, and its intersection with the celestial sphere is called a *vertical circle* or, simply, a *vertical.*



Of these, the vertical which passes through the east and west points, and is perpendicular to the meridian, is called the *prime vertical.*

Let ZST be a vertical passing through a star S, whose diurnal path is the circle $ASMA'$. It is obvious that the position of the star will be known, when we know the angle MZS between the meridian and the vertical, and the arc

TS which measures the angle TOS between the star and the horizon.

The angle MZS, or its equivalent the arc HT, is called the *azimuth*, and is usually reckoned from the south towards the east or west when the north pole is above the horizon, and from the north in the other case.*

The other angle TOS is called the *altitude* of the star. The angle ZOS, the complement of the altitude, is called the *zenith distance*, and may replace the altitude as one of the coordinates of the star's position.

14. The zenith distance diminishes as the star approaches the meridian; for, if SP be joined by an arc of a great circle, the spherical triangle ZSP will have the two sides SP, PZ constant while the angle P diminishes, therefore the third side ZS also diminishes, and the star attains its greatest altitude in the meridian. It is then said to *culminate.* Those stars which never set will cross the meridian again below the pole, and have there their least altitude.

The *culmination* of a celestial body is the instant of its attaining its greatest altitude. In the case of a star this coincides with the meridian passage, but for a body whose distance from the pole is changing the culmination may take place a little before or a little after. See Art. 215.

Declination and Right Ascension.

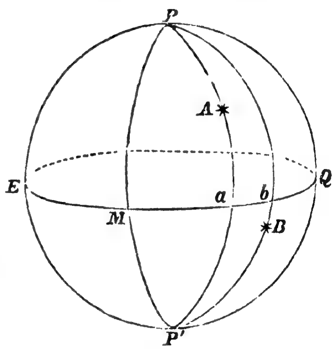
15. The altitude and azimuth, which thus define the position of a heavenly body, refer to one particular instant and one particular place of observation. Not only are these elements undergoing constant and rapid changes, owing to the diurnal motions, but they also differ, as we shall pre-

* Sometimes the supplementary angle PZS is called the Azimuth, but all ambiguity is avoided by specifying the point from which it is reckoned: thus N. 80° E. is the same as S. 100° E.

sently find, at the same instant, for observers at different places. We may, however, by referring the places of the heavenly bodies to the equator instead of the horizon obtain elements entirely independent of the observer's position; and, as far as the fixed stars are concerned, nearly independent of the time; their very slow change to which we shall have to refer hereafter amounting to only a few seconds of arc in the course of a year.

The *declination* of a star is its distance from the equator measured by the arc of great circle which passes through that star and the pole. This great circle is called the star's *declination circle*, and accompanies the star in its diurnal course.

The declination is either north or south, according to the side of the celestial equator on which the star is situated.



Thus if EQ be the celestial equator, P the north pole, P' the south pole, A, B two stars whose declination circles are $PAaP'$, $PbBP'$, then Aa, Bb are the declinations of these stars, the first being north and the second south.

The *polar distance* is the complement of the declination, therefore PA and PB are the north polar distances of the two stars, the one being less, the other greater than 90°.

16. The *right ascension* of a star is the angle made by its declination circle with that of some determinate point in the celestial equator. It is measured by the arc of the equator intercepted between them, reckoned eastward through 360°.

Thus if M be the chosen point, PMP' its declination circle, then the right ascension of the starts A and B will be Ma,

Mb respectively. The difference of right ascension will be *ab*, which is independent of *M*.

Since the right ascension and declination of a star are coordinates of its position at the time, a register of these, together with their annual change, if any be found, will enable us to identify a star once observed. Such a register is called a *Catalogue of Stars*, and its correctness is of the highest importance in Astronomy. The delicate instruments and means of observation, which we shall have to describe hereafter (Chaps. III., IV., V.), are specially intended for the verification and extension of this register.

When very great accuracy is not required, the register may be a globe on which the circles corresponding to the equator, and the different declination circles and parallels are traced at certain intervals apart, and the stars are marked on this globe in positions corresponding to those they occupy in the celestial sphere.

Instead of a globe a plane surface may be used, on which the positions are mapped according to certain rules of projection. This, though a less faithful representation of the heavens than the globe, will be more convenient for many purposes.

Many of the stars have received particular names, but their number precludes the possibility of doing this for all of them,—so the ancients very early divided them into groups, or constellations, as they are called, to which they attached names in some cases suggested by fanciful resemblances to figures of men and animals, but in others in a very arbitrary and confusing manner. The stars of each constellation are named in order of brilliancy by letters of the Greek Alphabet attached to them, as, α *Tauri*, β *Orionis*, &c.

We must leave the particular declination circle, whence the right ascensions are measured, to be chosen hereafter. For reasons to be then explained, it will be convenient to take it not passing through any particular star, but through

a point which, though partaking of the general diurnal motion, is really altering its position with respect to them, although at a rate so slow that the whole amount of change is only a few seconds annually.

Sidereal Day, Sidereal Time, Hour Angle.

17. The uniform revolution of the whole system of stars around the polar axis takes place from east to west, and is accomplished in what is called a sidereal day. This day is about 4 minutes shorter than an ordinary day, and clocks adjusted to keep sidereal time are called sidereal clocks. The whole day is divided into 24 hours, and the sidereal clock reckons onwards from 1.2.3....to 24.

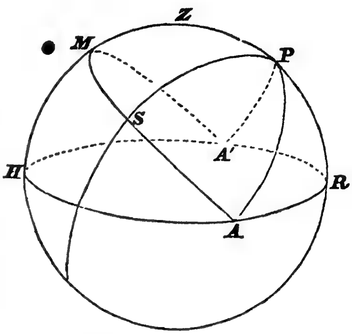
The sidereal clock is so adjusted as to mark 0 h. 0 m. 0 s. when the selected point (Art. 16), from which the right ascensions are reckoned, crosses the meridian of the observer. This point is called the *first point of Aries*, so that the correct definition of a *sidereal day* is "the interval between two consecutive transits of the first point of Aries;" and the *sidereal time* at any instant is "the number of sidereal hours, minutes, &c., since the last preceding transit of the first point of Aries."

It will be at once obvious that the different stars will succeed each other across the meridian in the order of their right ascension, and that, on account of the uniformity of the motion, the right ascension of each star, when reduced to time at the rate of 24 hours for 360°, or 1 hour for 15°, will give the sidereal time when that star comes to the meridian.

18. The angle which the declination circle of a star makes with the meridian, viz. the angle SPZ, is called the *hour angle*.

The hour angle SPZ and the polar distance SP may be

employed as coordinates of the star's position on the celestial sphere of the observer instead of the altitude and azimuth.

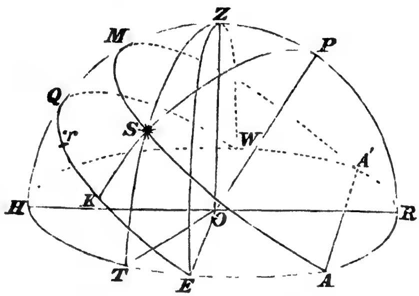


A knowledge of the hour angle of a star will, on account of the uniform rate at which it varies, at once give the time the star will take to reach the meridian if it be on the east side of it, or the time elapsed since it crossed the meridian if it be on the west side.

When a star is in the meridian one-half of its visible path is accomplished. Thus if AMA' be the parallel described by the star, A, A' being the points of rising and setting respectively, $HZPR$ the meridian, the two spherical triangles APH, $A'PH$ are right-angled at H, have the side PH common, and $PA = PA'$ since the star's polar distance does not change, therefore the hour angle APM at rising equals the hour angle $A'PM$ at setting.

For the convenience of reference, most of the terms introduced in this first chapter are here collected, and their connection shewn on a single figure.

Observer's celestial sphere.



O...Observer.
$REHW$...Horizon.
Z...Zenith.
S...Star.
AMA'...Parallel of star.
OP...Polar axis.
P...North pole.
HZR ..Meridian.
HR...Meridian line.

R...North point.
H...South point.
ZST...Vertical of star.
WZE...Prime vertical.
E...East point.
W.. West point.
EQW...Equator.
PSK...Declination circle of star.
♈..." First point of Aries."

The position S of a star at any instant may be determined by either of the three following systems of coordinates, viz.

(i) ST, the altitude, (or SZ, the zenith distance), and
HT, the azimuth;

(ii) SK, the declination, (or SP, the north polar distance), and
♈K, the right ascension; or

(iii) SP, the north polar distance, and
SPZ, the hour-angle.

CHAPTER II.

THE EARTH.

19. We have already said that a change of inclination of the polar axis to the horizontal plane will usually accompany a change of the observer's station. We shall now enter more fully into the circumstances of these changes, shewing how they are explained by the globular form of the earth, and also in what manner they will enable us to arrive at a knowledge of its size.

Axis of the Earth, Terrestial Meridian, Equator.

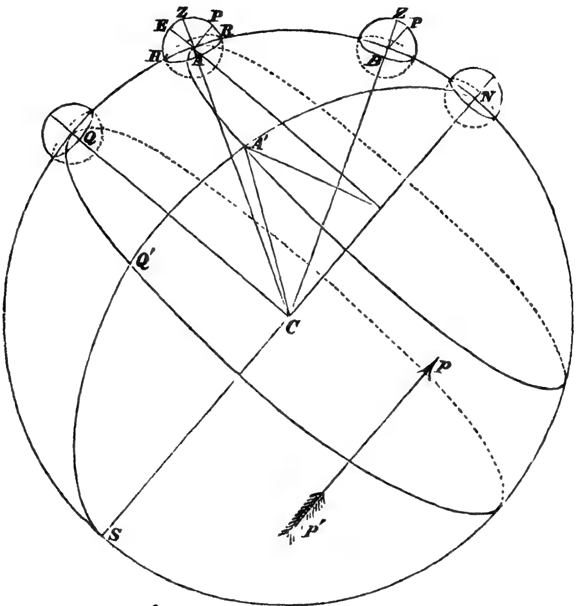
20. Let the large sphere in the accompanying figure represent the earth, C the centre, and let A be the first station of the observer, $p'p$ the constant direction of the polar axis, and the small sphere round A the observer's celestial sphere.

AP drawn parallel to $p'p$ will determine the pole P, the tangent plane HAR at A will determine the horizon, and the zenith Z will be found by producing CA to meet the observer's celestial sphere.

Let HR be the meridian line, then PAR is the angular elevation of the pole above the horizon, or the altitude of the pole, and ZAP its zenith distance; and if we draw the diameter SCN of the earth parallel to $p'p$, the zenith distance of the pole at A will also be measured by the angle ACN.

Since the meridian plane at A contains AP and passes through C, it will contain the diameter SCN; its intersection

with the surface of the earth will therefore be the great circle *SAN*.



21. Let the observer travel along this great circle towards the elevated pole to some second station *B*.

The pole *P* being determined as before by means of a parallel *BP* to *p'p*, and the verticle line *CBZ* giving the new zenith *Z*, we see that the zenith has approached the pole by an angle equal to the difference between *ZAP* and *ZBP*, or between *ACN* and *BCN*, that is, by the angle *ACB*.

By an observer ignorant of, or not attending to, the curvature of the earth, the change would be attributed to a motion of the pole which, carrying with it the parallels of the stars, would seem to have moved towards the zenith by that quantity. This way of considering the motion is perfectly adequate to a mere description of the phenomenon,

and, with this understanding, we shall occasionally make use of it.

If we refer to the figure (p. 8) we shall see that the effect of an approach of P to Z will be that the parallels of the stars will make a smaller angle with the horizon, the number of circumpolar stars will increase, and some of those which before remained a short time above the horizon will now never be seen.

22. That diameter SN of the earth which is parallel to the constant direction of the line about which the stars revolve is called the *axis of the earth*, and the points N and S the *poles of the earth.*

A *terrestrial meridian* is any great circle whose plane passes through the axis of the earth.

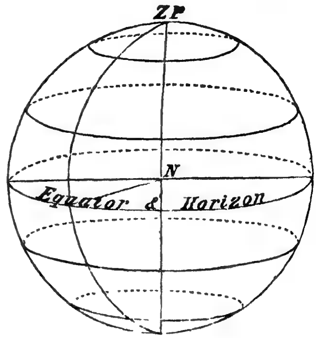
This definition holds when we consider the earth as a sphere or as a figure of revolution; and, in that case, the various stations A, B, &c., along the curve $SABN$ will obviously have their meridian planes coincident with the plane $SABN$. But, supposing the earth not to be a figure of revolution we shall give the following definition: "A terrestrial meridian is the locus of all points on the surface which have their meridian planes parallel." The curve will not necessarily be a plane curve.

The great circle QQ', whose plane is perpendicular to the earth's axis, is called the *terrestrial equator*, and all small circles parallel to the equator as AA' are called *parallels.*

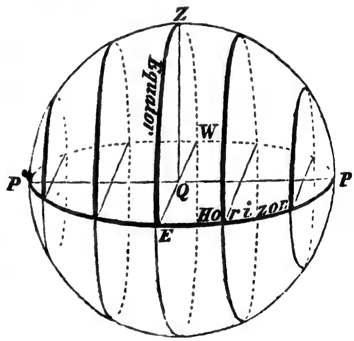
23. Let the observer continue his journey along the same meridian till he reaches the pole N. There Z and P will coincide, and the diurnal paths of the stars will be perpendicular to the vertical line, and therefore parallel to the horizon; the horizon itself will coincide with the equator. No star will ever rise or set, and the visible heavens will be confined to those portions which are on the same side of the equator as the elevated pole.

The observer has then what is sometimes called a *parallel sphere*, the different circumstances of which will be obvious

from the examination of the figure. There is no definite meridian, since the zenith and pole coincide, or rather every great circle through the zenith is a meridian. There will be no east or west line, nor in fact any other direction but south, supposing N to be the north pole.



24. If instead of advancing from A (fig. p. 18) towards the elevated pole, the observer travels in the exactly opposite direction, a contrary effect will be produced,—the pole will gradually recede from the zenith, and when the traveller reaches the terrestrial equator at Q, the poles P, P' will be in the horizon, and the zenith will be a point of the celestial equator. The parallels of the stars will all cut the horizon at right angles and be bisected by it, so that half their course will be above, and half below—every part of the heavens becoming visible in each revolution. The sphere is then called a *right sphere*.



25. Continuing his journey in the same direction along the meridian from Q towards S, the other pole will now become the elevated pole, and the stars round it the circumpolar stars. The sequence of phenomena being precisely the same as on the other side of the terrestrial equator, the south pole gradually approaching the zenith, and the parallels

of the stars about the north pole disappearing one after the other below the horizon.

26. The observer has hitherto been supposed to move along the same terrestrial meridian. Let us now take him to a new station A' on a different meridian $NA'Q'S$, and suppose A' to be on the same circle through A parallel to the terrestrial equator.

The two radii CA, CA' make equal angles with the axis of the earth, and therefore the zeniths of the two places A, A', though having different directions in space, will be equally distant from the pole of each observer's celestial sphere (Art. 20); but the phenomena of diurnal motion depend on the altitude of the pole; and consequently the very same phenomena will be perceived at A' as at A, though not at the same time; a star which passes through the zenith of A will have passed through the zenith of A' some time before, and through the zeniths of all places in the parallel $A'A$ in succession.

The horizons of the two places will also intercept different parts of the heavens at the same instant, but the very same portions will successively, and in the same order, present themselves.

Magnitude of the Earth.

27. We have seen (Art. 21) that the angle ACB subtended by the arc AB of a meridian is equal to the difference between the zenith distances of the pole at the two stations. Hence, assuming the observer to have measured these zenith distances, and also the distance travelled, he will know the arc AB and the angle at the centre, and from these the magnitude of the radius may be calculated.

Instead of the change of zenith distance of the pole he may take the change of *meridian zenith distance* of any star; for, since the star's angular distance from the pole is constant,

the alteration in the meridian zenith distance of the former will be equal to that of the latter.*

The length of the earth's radius will by this means be found to be something less than 4000 miles.

Latitude and Longitude.

28. The different places on the surface of the earth are distinguished from one another by their latitude and longitude.

The *geographical latitude* or simply the *latitude* is the angle between the zenith and the celestial equator at the place, or, which is the same thing, the angle between the vertical line and the plane of the equator. Thus at A (fig. p. 18) the latitude is the angle ZAE.

The *geocentric latitude* of a place is the angle subtended at the centre of the earth by the arc of meridian intercepted between the place and the equator. Thus AQ being the meridian of A, the geocentric latitude of A will be the angle ACQ.

Considering the earth as a sphere the angle ZAE will be equal to the angle ACQ, and the geographical and geocentric latitudes will be the same; but the distinction will be essential if we find the spherical shape not to be the true one (see chap. XVI).

29. Places on the equator QQ' have latitude 0°, and the latitudes increase from 0° to 90° on each side of the equator, being reckoned north or south, according as the place is towards the north or south pole. The latitudes of two places A, A' on the same parallel will be the same.

30. The angle PAZ, the complement of ZAE, is called the *co-latitude*.

* This method was employed by Eratosthenes (230 B.C.), except that he used the meridian zenith distance of the sun instead of that of a fixed star.

It will be useful to remark that PAR being the complement of PAZ will be equal to ZAE; that is, *the elevation of the pole above the horizon is equal to the latitude of the place.*

31. The *longitude* of a place is the angle made by its meridian plane with some one fixed meridian plane arbitrarily chosen, and is measured by the arc of equator intercepted between these two meridians.

The longitudes are reckoned from the fixed meridian through 180° on each side, one being east and the other west.

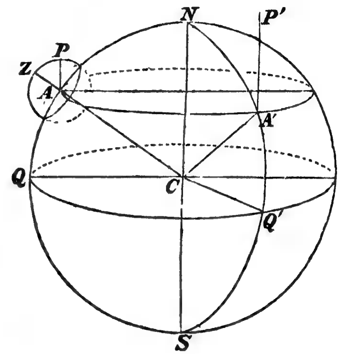
The Phenomena of Diurnal Motion explained by a Rotation of the Earth.

32. We have so far suppposed that the motion of the stars is a real motion, and that the earth is the fixed stationary body which it appears to us to be. We know, however, that the appearances of motion are frequently deceptive, and that it is often hard, when two things are in relative motion, to determine whether either of them, and, if either, which of them is absolutely at rest. Thus, when we are in a railway carriage moving smoothly along, the houses and trees appear to move in the opposite direction; and it is only our memory and our reason which tell us that it is we, and not they, who change places. But, on arriving at a station whence other trains are departing, it is often difficult to say, when looking at these from our own carriage, whether we ourselves have stopped and the other trains have commenced moving, or whether the motion is entirely ours, or partly ours and partly theirs.

33. Accordingly, let us now suppose the stars to be stationary, and examine what kind of motion must be attributed to the earth, in order that, on this hypothesis also, the same appearances may be produced as we have already

observed and recorded on the hypothesis of a stationary earth and moving stars.

Suppose the earth to turn uniformly about its axis *SN* from west to east, or in the opposite direction to that of the diurnal motion of the stars; and suppose it to accomplish its revolution exactly in that period which we found to be common to all the stars.



An observer at *A* will, after a certain interval, be carried to *A'*. His radius *CA'* will make with *CN* the same angle as before, and therefore the zenith of his celestial sphere, determined by the prolongation of *CA'*, will be at a constant angular distance from the point found by drawing *A'P'* parallel to *CN*. But *A'P'* will be in the meridian plane *NA'Q'*, which, turning with the earth, always passes through the same places on its surface; and therefore *A'P'* will not only always point to the same place in the heavens, but will also retain a fixed direction relatively to the observer and the terrestrial objects around him.

This is exactly the phenomenon observed, and the points of the celestial sphere determined by this fixed direction we have called the celestial poles.

Again, the stars which were in the meridian plane *NAQ* of the observer when he was at *A* will no longer be so when he has been carried to *A'*; but, on account of their immense distance, they will all be found in a plane through *A'P'* parallel to *NAQ*; and they will therefore be on the west of the observer's present meridian plane, which has become *NA'Q'*, the angle between the meridian and the great circle in which they lie being precisely the angle *QCQ'* through which the earth itself has turned; and, as we suppose the

earth to turn uniformly, the stars will appear to move towards the west at a rate which also is uniform.

34. Hence, whether we assume the stars to be in motion as one connected system about an axis through the earth at rest, or suppose the stars stationary and the earth to revolve in the opposite direction with the same angular velocity about this same axis, there is nothing in either supposition inconsistent with the appearances presented, and we have so far nothing to guide us in our choice of the explanation.

If the stars move, their declination circles will, one after the other, cross the meridian of the observer. If it is the earth that moves, then the meridian plane of the observer, carrying his zenith and horizon with it, will travel from west to east across the stars, and coincide with each declination circle in turn, the zenith during the revolution retaining the same angular distance from the pole, and therefore describing that small circle of the celestial sphere whose declination equals the latitude of the place.

Although the solution of problems concerning the apparent positions of the heavenly bodies will be correctly obtained on either supposition, and convenience alone need influence our selection, we may proceed at once to shew which of these two suppositions is the more probable.

Arguments in favour of the Earth's Rotation.

35. 1°. If we suppose the earth to turn about its axis in 24 hours, places at the equator will move through something like 25000 miles in that time, or about $\frac{2}{7}$ of a mile in a second. This velocity is great, but the other hypothesis will give to the stars, which we have reason to suppose to be bodies of enormous magnitude compared with the earth,

velocities infinitely greater—velocities which can only be reckoned by millions of miles in a second.

2°. The uniform period in which all the stars perform their circuit is, as we have seen, easily explained if the earth revolve; but the stars must have some *rigid* physical connexion if the motion be theirs, as it is beyond the limits of probability that an immense number of unconnected bodies should all describe circles of various dimensions in exactly the same time.

That the stars are not so connected is shewn by what are called double stars, which alter their relative position as if revolving one round the other: moreover the sun, moon, and planets partake of this general diurnal motion and move independently of the stars.

Simplicity of explanation is therefore manifestly in favour of the earth's rotation.

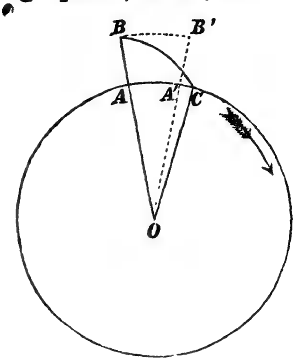
3°. An argument may also be drawn from the shape of the earth. Our rough measurements have told us that it is spherical; but a more exact determination will shew it to be an oblate spheroid, whose equatorial diameter is some 26½ miles longer than its polar diameter. This flattening at the poles is just the kind of effect that would be produced by rotation in a fluid, or semi-fluid mass, such as the earth is supposed to have originally been.

4°. The observations made on the sun and planets shew that all these bodies, several of which are much larger than the earth, have a motion of rotation about axes through their centre.

Proofs of the Earth's Rotation.

36. 5°. But we are not left to rely on probabilities only; proofs of the earth's rotation can be obtained by direct experiment.

Let OA be a radius of a revolving sphere, B a point in OA produced. Suppose a body let fall from B to be attracted towards O. The body will reach the sphere at some point C, describing as it falls a curve BC, because at B it has a velocity of projection perpendicular to OB.

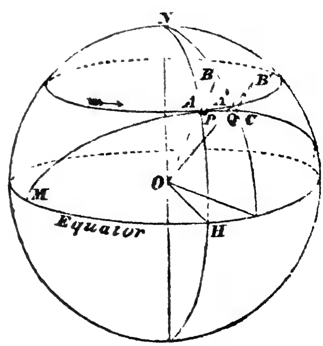


Let B' be the point which it would have reached in the same time if not let fall. Join OB' cutting the sphere in A' the new position of A. Then, the force being central, and the velocity and direction of motion the same at B for both paths BB' and BC, the area OBC will be equal to the area OBB'; therefore C will fall beyond A'—the magnitude of $A'C$ depending on the height AB and on the angular velocity.

If then the earth be at rest, the verticle BA, as shewn by a plumb line, is the direction which a falling body will pursue; but if there be any rotation of the earth from west to east, the body will fall to the eastward of the foot A' of the plumb line $B'A'$.

The preceding investigation applies to an experiment supposed to be made at the equator; for, we have assumed the rotation to be about an axis perpendicular to the plane OBB'.

Let us next consider A a place in north latitude, and let the rotation take place, as indicated by the arrow, from west to east.



A will describe the parallel AA', whereas the body let fall from B in OA produced will be moving at right angles

to the meridian plane NAH, and be acted on by a force towards the centre of the earth; consequently its path BC will be in the plane of the great circle MAC, which touches the parallel in A. Therefore C should be, not only east of A' as before, but also south of it.

But BA and $B'A'$ will no longer be the directions of the plumb line; for the rotation about NO will cause the line to diverge in the direction BP, south of BA, in such a manner that the earth's attraction and the tension of the string acting on the plummet P, may have a resultant perpendicular to NO.

If λ be the latitude of the place, ω the angular velocity of the earth, r its radius, f the resultant acceleration,

$$f = r\omega^2 \cos\lambda,$$

and if h be the height BP, and mg the tension of the string, m being the mass of the plummet,

$$AP = h \sin ABP = h \frac{f \sin\lambda}{g} = \frac{rh\omega^2 \sin 2\lambda}{2g}.$$

Now, let the meridian NA' cut the great circle AC in Q, then $A'Q$ is approximately the southerly deviation of the falling body; and if t be the time of falling, we have $ANQ = \omega t$. Therefore, from the right-angled triangle NAQ,

$$\tan\frac{NA}{r} \cdot \cot\frac{NQ}{r} = \cos\omega t,$$

$$\frac{\sin\dfrac{NQ - NA}{r}}{\sin\dfrac{NQ + NA}{r}} = \frac{1 - \cos\omega t}{1 + \cos\omega t},$$

$$\frac{\sin\dfrac{A'Q}{r}}{\sin 2\lambda} = \tan^2\frac{\omega t}{2},$$

$$A'Q = \frac{r\omega^2 t^2}{4} \sin 2\lambda \text{ approximately},$$

$$= AP, \text{ since } h = \tfrac{1}{2}gt^2,$$

therefore, when B reaches B', the plummet P will be at Q; that is, there will be no southerly deviation of the falling body relatively to the actual vertical BP.

The resistance of the air has been neglected, but when this is taken into account the conclusion is still the same.* If, on the one hand, the time of descent is increased which would tend to make the body fall nearer the equator, the impulses it receives from the particles of air, which are describing circles about the axis, tend to bring it back towards the pole, and one effect is found to counteract the other.

The easterly deviation measured by QC may be very approximately obtained as follows: the area OBC described by the falling body under the action of the central force at O will be approximately equal to OBB'. Take away the equals AOA' and AOQ, and we have

$$\text{area } BAA'B' = QOC + ABC,$$

but the curve BC is approximately a parabola, therefore

$$h.AQ = \tfrac{1}{2}r.QC + \tfrac{2}{3}h.(AQ + QC),$$

$$QC\left(\frac{r}{2} + \frac{2h}{3}\right) = \tfrac{1}{3}h.AQ,$$

$$\tfrac{1}{2}r.QC = \tfrac{1}{3}hr\omega t \cos\lambda \text{ neglecting } \frac{4h}{3r} \text{ compared with 1},$$

$$QC = \tfrac{2}{3}h\omega t \cos\lambda.$$

The experiments which have been made with the greatest care by different persons all tend to confirm the results here obtained, and the amounts of easterly deviation, though small, agree closely with the theoretical values calculated on the supposition that the earth revolves on its axis once in 24 hours. In some of the experiments a small

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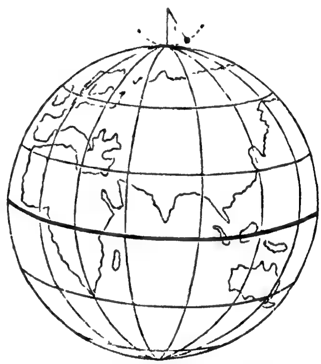
* Vide *La Place Méc Cél.* vol. I. p. 98, and vol. II. p. 104. Also *The Earth and its Mechanism*, by H. Worms, F.R.A.S., F.G.S., which contains a full account of the various experimental proofs of the rotation of the earth.

northerly and in others a small southerly deviation was observed; but these, like the small differences between the observed and the calculated values of the easterly deviation, are due to errors which cannot be avoided, such as currents of air or slight vibrations of the ball when set free.

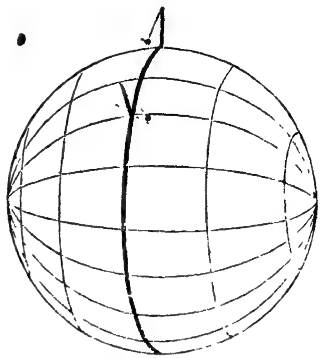
37. 6°. Two experiments devised a few years ago by Mons. Foucault are still more satisfactory, because the effects are more observable.

The first is known as the pendulum experiment, and may be explained in the following manner: Assume the earth to be in motion with an angular velocity (ω) from west to east; and, firstly, suppose a pendulum suspended over the north pole of the earth and there made to oscillate. There would be no force acting on the pendulum out of the plane of oscillation, and therefore that plane would retain a fixed position in space; and, the earth revolving under it from west to east, the different meridians would, one after the other, coincide with the plane of oscillation; the apparent effect to a person at the pole, and not aware of his own motion, being a gradual shifting of the plane of oscillation from east to west; a complete revolution being accomplished in one day $\left(\frac{2\pi}{\omega}\right)$.



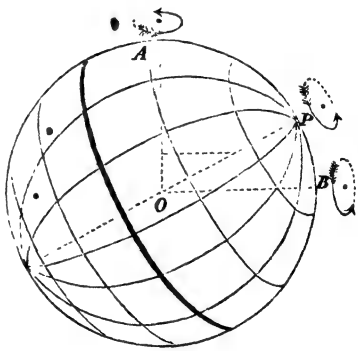
Next, suppose the pendulum to be placed at the equator; the bob of the pendulum before the oscillations begin will partake of the general motion of rotation round the axis of the earth parallel to the plane of the equator, and in

whatever direction the pendulum be set in motion, there is nothing to cause a disturbance of the plane of oscillation relatively to the horizontal plane, since all parts of the horizontal plane in the immediate vicinity of the bob of the pendulum will also have the same common velocity of rotation.



This will not be the case at a place off the equator, because there those parts of the horizontal plane just under the pendulum which are nearest to the pole will have less velocity than those nearest the equator, and an apparent motion of the plane of oscillation with respect to the plane of the meridian will be the consequence.

Let A be a place in the north latitude λ. The angular velocity ω about the axis OP may be resolved into two; the first $\omega \cos AOP$, or $\omega \sin\lambda$ about OA, and the other $\omega \cos POB$ about an axis OB perpendicular to OA (Routh's *Rigid Dynamics*, p. 120).



Their effects may be considered separately; the rotation about OB will produce no disturbance since A would be like a point in the equator with respect to that axis; but the rotation about OA will have the same effect as in the first case considered—that of a pendulum at the pole. The angular velocity being $\omega \sin\lambda$, the time of a complete revolution will be $\frac{2\pi}{\omega \sin\lambda}$, but $\frac{2\pi}{\omega}$ is one day; therefore

the time will be 24 cosecλ hours. In the latitude of Cambridge this will be about $30\frac{1}{3}$ hours.

Now a pendulum suspended in this manner will not oscillate long enough to make a complete circuit, but it will do so during a sufficient time to enable us to verify that there is a displacement of the plane of oscillation, and, by a simple proportion, we can ascertain that if it continued at the same rate it would agree with the result obtained.*

If the earth did not rotate we should have no means of explaining this result.

38. The other experiment, imagined by Mons. Foucault, is independent of the earth's attraction.

If a body, symmetrical with respect to an axis through its centre of gravity, be made to rotate about that axis, the centre of gravity alone being supported and the axis free to move in any manner round it, it may be shewn that no change will take place in the direction of the axis in space, provided no force but gravity act upon the body (Routh's *Rigid Dynamics*).

By an ingenious contrivance, called a gyroscope, Mons. Foucault obtained this permanent axis, and verified that only when parallel to the earth's axis did it retain a permanent direction with respect to surrounding objects. In all other positions it moved just as the stars seem to move, and would, in fact, if pointed towards any particular star, have continued to point to this star during the whole time that its rotation lasted. The stars therefore have permanent

* For the success of the experiment several precautions must be observed. The pendulum must be as long as possible in order that the angle of vibration may be small, and the motion should originate by an impulse on the bob in the position of rest; for, if the motion is obtained by drawing the pendulum out of the vertical, and when perfectly steady, setting it free, the projection of the bob on the horizontal plane will be approximately a straight line, but really an elongated ellipse, whose major axis, however, would have the east to west displacement mentioned above.—(*Quarterly Journal of Mathematics*, 1858).

directions in space, and thus we have another independent proof that the apparent diurnal motion of the stars belongs in reality to the earth.

39. We shall, however, continue to speak of the rising and setting of the heavenly bodies and of their crossing our meridian, although we now know that strict astronomical language would require us to say, in the one case, that the plane of the horizon is sinking below, or rising above, the body; and, in the other, that the plane of the meridian is crossing it. But, as already stated in Art. 34, when concerned with questions which involve only the apparent directions, either supposition may be made use of.

CHAPTER III.

THE OBSERVATORY.

40. We must interrupt our observations to explain the construction and use of some of the instruments by means of which the coordinates of the stars' positions may be ascertained with extreme precision.

The ancient astronomers generally divided their instruments to 10′, and although some observations are found recording much smaller sub-divisions, they are not entitled to much confidence. The latitude of Alexandria, as determined by Ptolemy, differs by about a quarter of a degree from that which is given at present.*

Tycho Brahé (1570), whose instruments and methods of observation were greatly superior to those of former astronomers, carried the division of some of his instruments to every minute, and could still further estimate to 10″. This required instruments of a very large size, which consequently became unwieldy and liable to derangement from their weight. The precision besides was only apparent, for the telescope was not yet known, and his only means of pointing was by plain sights or projecting pinnules of metal, having slits in them, through which the star was observed. The uncertainty from this cause very far outweighed the supposed accuracy of the division. It may be said, that before the invention of the telescope the positions of the stars could scarcely be ascertained to within 5′.†

* Delambre *Astronomie Moderne*, I., 37.

† Flamsteed found that the fixed stars in Tycho Brahé's catalogue were generally 5′ or 6′ in error, and in some instances even more. (Baily's *Life of Flamsteed*, p. 125).

41. The invention of the telescope (1609) did not at once get over the difficulty. The first telescopes made were all on the principle of Galileo's, and they offered no means of fixing the direction of a star, since there is in them no position where cross-wires could be placed so as to be seen directly at the same time as the star.

The Astronomical telescope consisting of two convex lenses was suggested by Kepler (1611), but Gascoigne (1640) was the first to perceive its value in Astronomical observations. The common focus of the two lenses gave a place, within the instrument, where cross-wires of reference could be fixed and seen distinctly with the celestial object. He at once adapted telescopic sights to his instruments, and was the first who did so with success, by the help of these threads or wires, which enabled him to point the optical axis of the telescope with the greatest nicety.

Improvements in the means of dividing and graduating the instruments have since then gone on at a pace commensurate with the requirements of Theoretical Astronomy, and in the instruments now in use in all large observatories, the graduation is, by the aid of the micrometer (also an invention of Gascoigne's) and of the microscope, estimated to a fraction of a second.*

Clocks—The 'Balance' Clock.

42. Another event which largely contributed to the accuracy of observations, and was second in importance only to the discovery of the telescope, was the improvement effected in the means of measuring time by the introduction of the pendulum as a regulator of clock mechanism. The idea of employing the pendulum for this purpose was suggested by Galileo, who had remarked the apparent

* For a full and interesting account of the advance of practical astronomy, and of the labours of those eminent men to whom its rapid progress in modern times is due, see Grant's *History of Physical Astronomy*, p. 434.

isochronism of the vibrations; but Huyghens (1656) first succeeded in constructing a clock on this principle.

The element of time had seldom been employed by any observer, and by none with confidence; the clocks previously in use being liable to constant sources of irregularity.

43. The best form then known was the *Balance* clock, constructed on the following principle: A cord wrapped round a horizontal cylinder supported a heavy weight. The action of the weight tended to turn the cylinder, so that if not restrained an accelerated motion would be produced. But the cylinder was connected with a train of wheelwork in such a manner that the last wheel, a vertical one, was obliged to turn 60 times faster than the cylinder. To this wheel, called a crown-wheel from its shape, was applied the check or restraining power in the following manner:—A vertical rod called a *verge*, placed in front of, and nearly in contact with the wheel, was moveable about pivots at its extremities, and had two projecting pallets separated by a distance equal to the diameter of the wheel. Firmly fitted to the verge were two thin projecting arms carrying small weights, forming with the verge a kind of cross which was called the *balance*. These arms were placed near the top, so that when the verge turned upon its pivots they passed clear of the wheel. The pallets were so placed (their planes being 90° apart) that the highest tooth of the wheel met the upper pallet and, in its effort to turn, pushed it round, and with it the whole balance. As soon as the highest tooth had passed, the lowest tooth found itself opposed to the lower pallet, and the motion just before given to the balance was thus checked and stopped. A reverse motion now took place to allow this lowest tooth to pass, when the upper pallet again came into play, and so on alternately. Thus the wheel always turning the same way gave the balance an oscillating motion, which moderated

and regulated the velocity of the descending weight and of the cylinder. The revolutions of the cylinder served to measure time; its axis projecting outside of the framework was connected by other wheels and pinions, with hands indicating hours and minutes on a clock face. By shifting the weights along the cross arms of the balance, the rate of the clock could be increased or diminished.

Fig. 1 shews a balance clock which was put up in the palace of Charles V. of France, about the year 1370, by Henri de Vic, a Norman. The figure is taken from Berthoud's *Histoire de la mesure du temps par les Horloges*, Paris, 1802; but all those parts of the figure which do not refer to the action of the weight and balance have been omitted. It will be seen that the balance B is suspended by a cord M, in order to diminish the friction on the pivots. P and Q are the two pallets on the verge PQ, forming with the crown-wheel A what is technically called the *escapement*. The weight W acting upon the cylinder C turns the wheel F of 64 teeth. This wheel acts upon the pinion G which has 8 leaves, and therefore makes 8 revolutions for every one revolution of the cylinder. On the same axle with G is fixed the wheel H, which therefore revolves at the same rate as G. The wheel H has 60 teeth, and acts upon the pinion K which has 8 leaves; K therefore turns $7\frac{1}{2}$ times faster than H, or 60 times faster than the cylinder. Now, if the weights s, s on the arms of the balance be so adjusted, by trial, that the crown-wheel A which is on the same axle as K may make one revolution exactly in a minute, the cylinder will make one in an hour; and the axle D of the cylinder can be made to communicate this motion by another train of wheelwork (not represented in the figure) to the hour and minute hands of a clock face.

44. When the weight has run down, and it is necessary to wind it up again, a simple artifice allows of this being

done without disturbing the hands, or any of the wheels or pinions which connect the cylinder with the crown-wheel. The wheel F is rigidly attached to the axle D, but the cylinder is not. A ratchet-wheel (not given in fig. 1, but shewn in fig. 2) turning with the cylinder and rigidly-attached to it is in contact with F, and a click fastened to the face of F catches in the teeth of this wheel when the cylinder is turned by the action of the weight, thus producing motion in the whole train. But when the clock is being wound up, and the cylinder is turned in the opposite direction, the click slips over the teeth of the ratchet-wheel, and the cylinder turns alone.

45. Ingenious as was this clock, there were several defects in it which rendered it unfit for the delicate purposes of Astronomy. In the first place, it is obvious that the least irregularity in the size or position of the teeth of the wheel which acted upon the pallets would allow some of them to slip by more rapidly than others, producing a corresponding irregularity in the velocity of the descending weight. Again, the rapidity with which the oscillations of the balance were performed would depend upon the force with which the wheel pressed against the pallets, and upon the distribution of the mass of the balance relatively to its axis. This latter was affected by every change of temperature; and the former, not only by changes of temperature, but also by the addition of the varying weight of the unwound portion of the cord to the descending weight, however slight the chord might be. Tycho had some of these clocks, but for the above reasons he never relied upon them as elements of observation.

The Pendulum Clock.

46. Huyghens had the happy idea of substituting the pendulum for the balance in such a manner that the down-

ward motion of the weight was regulated by the isochronous* oscillations of the pendulum. The mode of escapement was the same as in the balance clock, but the verge with the two pallets was horizontal, as also the wheel which acted upon them. This will be easily understood from fig. 3, which is taken from the same work as the former (some parts being omitted as before).

The difference between the balance clock and that of Huyghens seems slight, but it is essential. In the balance clock the weight is the cause of and, to a considerable extent, commands the motion. In the pendulum clock the weight has no longer the same influence; the length of the pendulum regulates the duration of the oscillations, and any variation in the descending weight, or in the action of the intermediate wheels, may increase or diminish the extent of the arc of oscillation, but has only a very slight effect on its duration.

The weight, or some other source of power, is still necessary however to turn the hands and wheels which measure the time, and also to maintain the vibration of the pen-

* The oscillations of a rigid pendulum are not strictly isochronous, as was at first imagined by Galileo. Huyghens saw that in order to be so, the arc described by the centre of oscillation must be a cycloid, and not an arc of a circle, and his pendulum was constructed to produce this effect. The upper part terminated in two threads, which during the oscillations wrapped upon two cycloidal cheeks (as in fig. 4), and thus gave to the centre of oscillation its cycloidal motion. (See Parkinson's *Mechanics*).

In practice, however, the cycloidal pendulum was not found to give results so satisfactory as had been expected:—the theory which shewed the oscillations to be isochronous, contemplated a simple pendulum oscillating freely, and took no account of the pressure which resulted from its necessary connection with the escapement and wheelwork. It was, therefore, soon rejected for the common pendulum oscillating about a fixed axis. The oscillation of the latter would be isochronous, if the arc described remained always the same; but, it may be shewn, that the departure from isochronism will be inappreciable so long as the oscillations do not extend beyond 2° or 3° on each side of the vertical; and as a slight pressure or impulse is given to it each time to repair the loss of motion due to friction, &c., this tends to maintain the extent of oscillations the same, and therefore to produce strict uniformity.

The cycloidal pendulum was altogether abandoned after the invention of the anchor escapement described in p. 41.

dulum. This is done by causing the slight pressure on the pallets to be transmitted to the pendulum by means of the fork S, between the prongs of which, at T, the pendulum passes, and whose other extremity R is attached to the verge PQ. Without this, the pendulum would, sooner or later, be brought to rest by the resistance of the atmosphere and by friction.

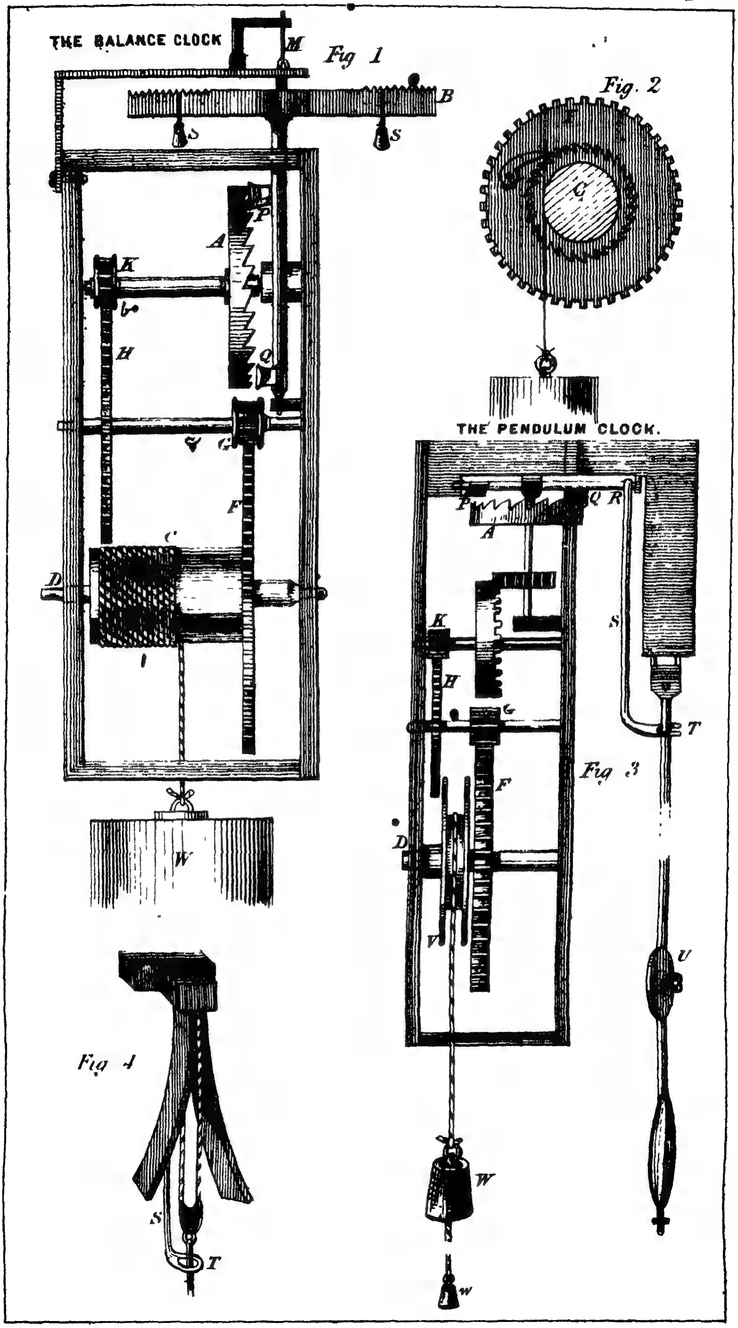
Instead of the cylinder, Huyghens substituted a pulley, over which the cord passed, with a large weight on one side and a small one on the other. The cord was hindered from slipping by covering the circumference of the pulley with sharp projecting points.

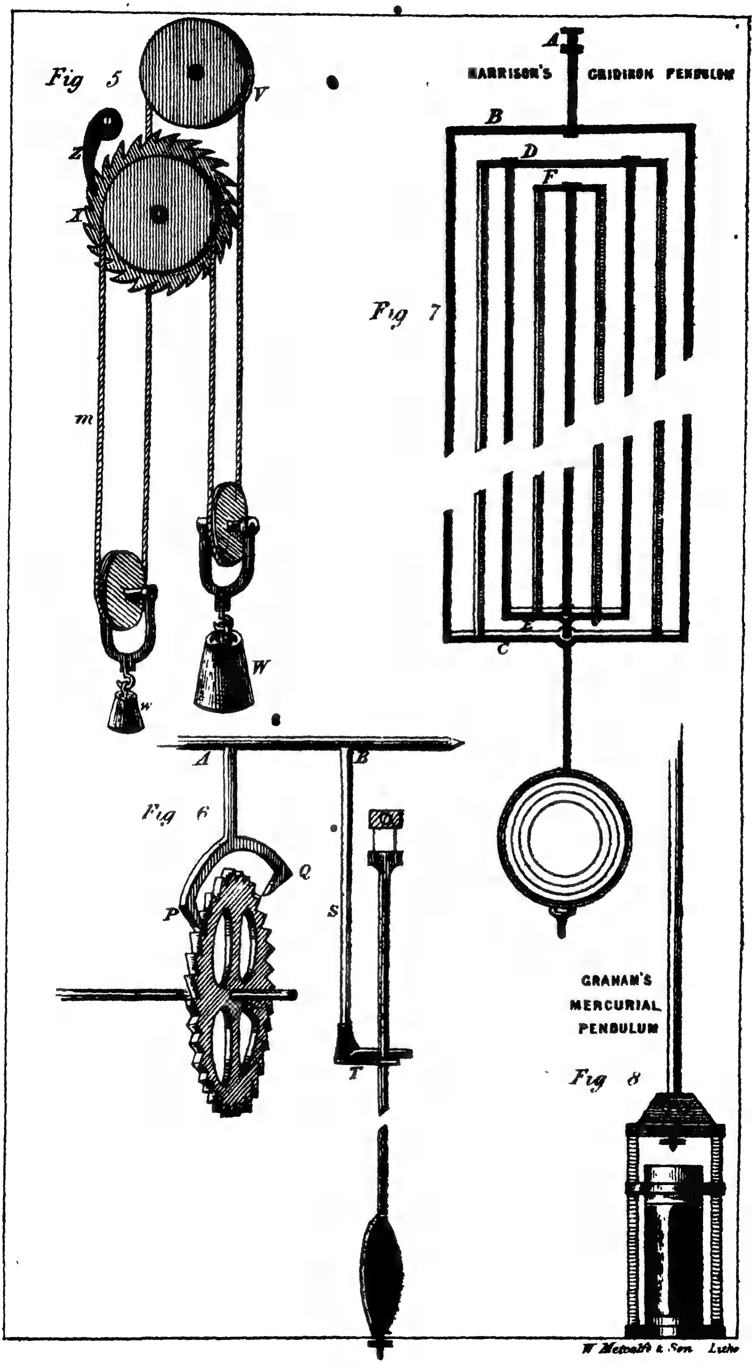
47. We have seen how, in the balance clock, the weight was wound up again without acting upon the train or the hands. The same artifice will clearly apply to the pendulum clock, but during the time of re-winding, the clock stops, and though the error would be unimportant among the other irregularities of the balance clock, it could not be overlooked in one intended for astronomical purposes. To Huyghens is due the following simple means of winding up the clock without interfering at all with its regular and uniform progress:—

Suppose V (fig. 5) to be the pulley of Huyghens' clock. The cord passing over it, instead of being attached immediately to the two weights, is an endless cord which passes under two smooth moveable pulleys from which the weights are suspended, and then over another pulley X fixed to the framework of the clock. This last pulley is rough like the first to hinder the cord from slipping. A click Z catches into the teeth of a ratchet-wheel in the circumference of the pulley X, so as to hinder it from turning in the direction of the heavier weight, but allows motion in the opposite direction.

When, by the slow and gradual turning of the pulley

To face p. 40


THE BALANCE CLOCK
Fig 1
M
B
S
S
P
A
K
b
H
Q
G
F
C
D
W
Fig. 2
C
THE PENDULUM CLOCK.
P
Q
R
A
S
K
H
G
T
Fig 3
F
D
V
U
W
w
Fig 4
S
T



W Metcalfe & Son Litho

V, the weight W has descended, and requires to be again drawn up, the hand must be applied to the cord at m. By pulling downwards, the weight W will be raised, and w lowered, with little, if any, alteration in the tensions of the strings on each side of V,* and therefore none whatever in the going of the clock.

48. Among the numerous improvements which have been made in the construction of clocks since that time, we shall only particularise a few of the more important.

The 'Anchor' Escapement.

The *anchor* escapement, so called from its shape, is a modification of the verge-and-pallet escapement, and was substituted for it by Mr. William Clement, a clockmaker of London, about 1680, though the celebrated Hooke claimed to have invented it as early as 1666.

Fig. 6 will explain the action of this escapement. The verge AB is horizontal, and the fork S with its prongs T are attached to it, and act upon the pendulum just as in Huyghens' clock; but the escapement wheel is vertical, and the pallets PQ are attached to a curved piece projecting from the verge, and in the same plane with the escapement wheel, of which it embraces a greater or less arc. With each swing of the pendulum, one of the two pallets catches into a tooth and stops the wheel; then, on the return swing, the tooth slips by and the wheel begins to turn, until stopped by the pallet on the other side, and so on alternately. The great advantage of this escapement was its requiring a much smaller arc of oscillation in the pendulum, and therefore securing much better the isochronism of the oscillations. (See note, p. 39).

* Any slight alteration will be due to the want of uniform velocity in the hand; for instance, at the beginning and end of each pull.

Compensating Pendulums.

49. The time of oscillation of the pendulum depends upon its length; it is obvious, therefore, that the expansions and contractions due to heat and cold must interfere with its uniform rate. Clocks will lose in summer and gain in winter. The amount of this loss or gain will vary with the material of which the pendulum is composed, because all substances have not the same expansibility; but the irregularity will exist in all. In Huygens' clock, *U* is a small weight, which, being moved along the pendulum, serves to correct the rate of the clock.

Harrison's gridiron-pendulum and Graham's mercurial-pendulum, both invented about the year 1726, are ingenious and valuable means of counteracting the effect of changes of temperature. They are *compensating pendulums* constructed by taking advantage of the unequal expansions of different substances, and so arranging combinations of them as to leave the distance from the centre of oscillation to the centre of suspension unaltered at all temperatures, the pendulum becoming thus self-adjusting.

50. The gridiron-pendulum consists of 5 steel and 4 brass rods, connected at top and bottom by cross pieces of brass *B*, *C*, *D*, *E*, *F*, as in fig. 7, where the steel rods are represented by dark lines.

The coefficients of expansion of steel and brass, *i.e.* the quantities by which any lengths of these metals must be multiplied in order to obtain the expansion for an increase of temperature of 1° centigrade, are*

steel ·0000107912,
brass ·0000187821,

or, in the proportion of 4 to 7 very nearly.

Now, on an increase of temperature, the cross piece *B* will be lowered by a quantity proportional to the length

* Biot's *Physique*, Paris, 1824, vol. I. p. 231.

of the steel rod which connects it with the point of suspension A. C will descend by the same quantity, and, in addition, that due to the lengthening of the steel rods from B to C. D is carried down with C, but the expansion of the brass rods connecting them will bring D up again through some space. The motion of E downwards will be the same as that of D, *plus* that due to the steel rods joining them, and so on.

The centre of the bob of the pendulum, which is connected with F by a steel rod passing freely through the cross pieces C and E, will therefore increase its distance from A by a quantity equal to the expansion of the steel rods, minus that of the brass rods. If, therefore, the total lengths of the steel rods and brass rods be as 7 : 4, or in the inverse proportion of their expansibilities, the distance between the point of suspension and the centre of oscillation will remain unchanged.*

51. In Graham's mercurial-pendulum (fig. 8),† a glass cylinder containing mercury is suspended by a steel rod, which supports the bottom of the cylinder.

The coefficients of linear expansion for 1° centigrade are

$$\text{mercury} \quad \cdot 0000600601 = m,$$

$$\text{glass} \quad \cdot 0000087572 = g.$$

Therefore the coefficient of expansion of mercury in a glass tube is

$$3m - 2g = \cdot 0001626659,$$

and that of steel is $\cdot 0000107912$.

* The centre of the bob is not necessarily, nor generally, the centre of oscillation; but in a seconds' pendulum, when the bob is heavy and the rods light, the distance between them will be small and sensibly constant.

† For an account of Graham's invention, see *Phil Trans.* 1726. See also Routh's *Rigid Dynamics*, 2nd Edition, p. 70, where a strict investigation is given of the relative dimensions of the parts of a Graham's pendulum, when the mercury is enclosed in a cast-iron cylindrical jar, into the top of which an iron rod is screwed.

These are in the ratio of 15 to 1 nearly, therefore if the cylinder of mercury have about $\frac{2}{15}$ the length of the rod, and a sufficient diameter to bring the centre of oscillation near the middle of the mass of mercury, the fall of the centre due to any expansion of the steel rod will be counteracted by its rise due to the expansion of the mercury.

52. In the gridiron and mercurial pendulums, the duration of an oscillation is made greater or less by moving the bob in the one or the cylinder in the other by means of screws which connect them with the rod; but as they may both be over-compensated or under-compensated for changes of temperature, Graham's will have an advantage over the other, because it admits of easy correction, which the astronomer himself can perform by merely withdrawing or adding more mercury.

Conical Pendulum, Spring Governor.

53. It will be obvious that the motion of the hands produced by the successive beats of the pendulum is intermittent, the advance being by jerks and starts. The exact termination of each second is well marked by the beat (supposing the pendulum to be a seconds' pendulum), but the subdivisions of the seconds can only be obtained by estimation. The perfection to which Astronomy has now attained requires that even these fractions of a second should be given accurately.

To obtain a continuous uniform motion, Huyghens suggested the use of a conical pendulum,* but it is only within the last few years that it has taken a practical form. The Astronomer Royal employs the rotation of a large conical pendulum to govern the motion of a clock. The inclination of the pendulum to the vertical soon adjusts itself, so that

* *Horologium Oscillatorium*, p. 157.

the resistance of the air and the friction of suspension are exactly balanced by the force which maintains the motion. A motion without jerks and sensibly uniform is the result.*

A few years earlier (1849), the Messrs. Bond, in America, invented the *Spring Governor*, "consisting of a train of clock-work connected with the axis of a fly wheel. It has an escapement wheel, into the teeth of which pallets play by the oscillation of a pendulum, as in ordinary clocks, the wheel being so connected with its axis by a spring as to allow the axis to move while the wheel is detained by the pallets."† The contrivance produces continuous and very approximately uniform rotation of the axis.

* For a detailed description, see *Appendix to Greenwich Observations*, 1856.

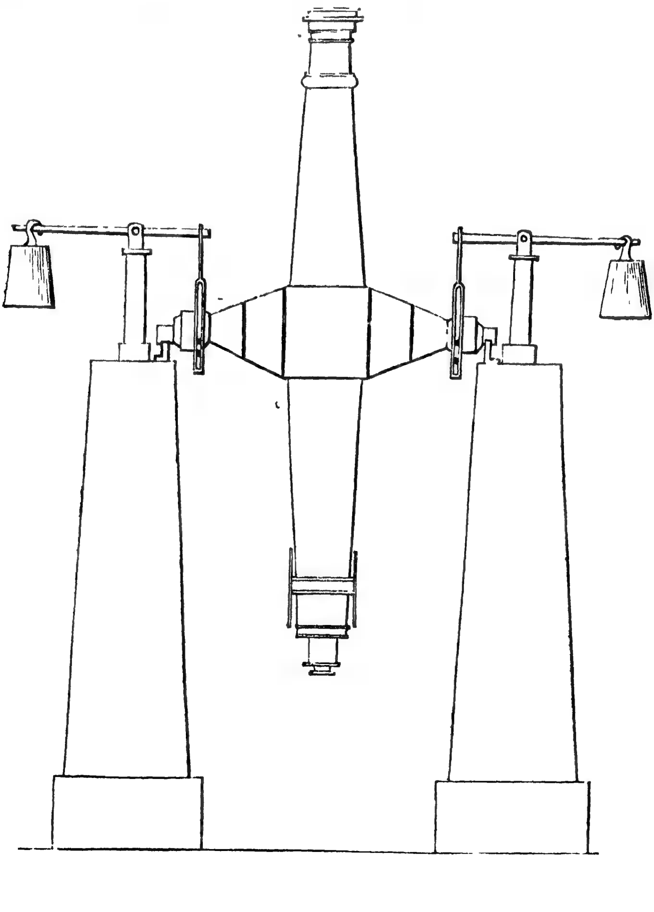
† Loomis' *Practical Astronomy*, p. 79.

CHAPTER IV.

THE OBSERVATORY CONTINUED.

The Transit Instrument.

54. THE transit instrument is one of the most important in an observatory. Its object is to determine the precise

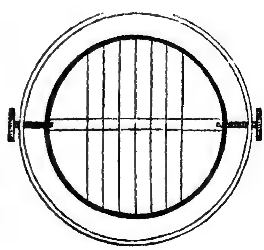


instant at which a celestial body crosses the meridian, and, for this purpose it consists of a telescope moveable about a fixed horizontal transverse axis of rotation. This axis points east and west, and the optical axis of the telescope is at right angles to it. As the whole instrument turns the optical axis never deviates from the plane of the meridian, and may be directed towards any point in it.

The fixed axis of rotation is obtained by having a strong tubular piece of metal whose ends terminate in cylindrical pivots exactly equal in size resting in sockets. These sockets, called *Y*s from their shape, are firmly fastened to two stone piers; but, for the purposes of adjustment, the one admits of a small vertical and the other of a small horizontal motion by means of fine screws. The geometrical axis of the two pivots is the fixed axis of rotation, and the framework of the telescope projects on each side from the middle part of the hollow tube.

In order to diminish the friction and wear of the *Y*s, the weight of the instrument is almost wholly counterpoised by two weights (see the figure) acting on levers over the piers, care being taken that sufficient weight remains on the *Y*s to ensure that the direction of the axis shall be determined by them.

The telescope used is an astronomical telescope, with an achromatic object-glass and a Ramsden's eye-piece. In the focus of the object-glass is placed a frame-work carrying, at equal intervals, five or seven spider lines or wires in vertical directions, intersected by two horizontal lines, between which the star is observed. The frame-work admits of various small motions for the sake of adjustment.



To render the lines visible at night, the light of a lamp placed on one of the piers is admitted through the hollow

pivot, and directed to them by an elliptic ring reflector placed diagonally at the junction of the axis and telescope. The quantity of light admitted may be regulated by a moveable plate, which enlarges or diminishes the aperture. A bright field and dark lines will thus be obtained; but when very faint stars have to be observed, it will be better to illuminate the lines and leave the field dark, which may be done by means of a small lamp fitted to an aperture in the telescope tube between the lines and the eye-piece.

The eye-piece used is the positive or Ramsden's.* This has the advantage, that it may be changed for one of a different power without disturbing the wires, which, being situated beyond the field glass, are entirely separate from the eye-piece. And the field glass, being plano-convex, offers a flat surface to the image, so that the wires and all parts of the field of view are distinct at the same adjustment.

55. The object of having several vertical wires is to secure greater accuracy in the observation. We may always expect a slight error in estimating the exact instant, as shewn by the clock, when a star appears to pass behind any one of the wires; but, by noting the times across each of the seven wires, and taking the mean, we obtain the time over an imaginary wire nearly, if not exactly, coincident with the middle one, and called the mean of the wires. As the error of estimation across the different wires is likely to be in excess for some and in defect for others, the probable error of the mean will be much less than the error of any single wire.

The *line of collimation* is the straight line joining the centre of the object glass, with the point of this imaginary vertical wire midway between the two horizontal ones.

* See Parkinson's *Optics*.

56. Supposing the instrument perfect both in construction and in adjustment, the line of collimation will be at right angles to the east and west line about which the instrument rotates, and will exactly describe the plane of the meridian; so that at the instant when any star or other object is on the mean wire it will be crossing the meridian. The manner of the observation is as follows:—As soon as the star enters the field of view, the observer writes down the hour and minute shewn by the clock; then, taking a second from the clock face and following the beats, he notes the instant when the star appears on the first wire, writes it down, and, continuing the reckoning, does the same for each of the seven wires. The mean will be the time of transit as shewn by the clock. If the star is not on one of the wires exactly at a beat of the clock, the observer must judge of the distances of the wire from the positions of the star, one on each side, when two successive beats are heard, and thus estimate the fraction of a second. A practised observer will estimate to tenths.

Of late years a method of observing has been employed by the American astronomers, and is coming into general use, by which the instant of crossing each wire is more accurately determined: A cylinder or drum, covered with a sheet of paper, is made to revolve about its axis with a uniform and jerkless motion, by connecting it with clock-work having either a conical pendulum or a spring governor for its regulator (Art. 53). At the beginning of every second, the clock interrupts an electric circuit, and a corresponding dot or mark is made on the paper. The cylinder has a slow motion in direction of its length, so that the marks made, which are generally about an inch apart, arrange themselves without confusion in a spiral curve on the paper. When the instant arrives which the observer wishes to record, he presses a button near his hand, and an instantaneous mark is made on the paper, which, by its position relatively to the adjacent

seconds' marks, enables him to measure the fraction with very great accuracy. As the mark made is a permanent record of the observation, the measures need not be taken till after the observation is wholly concluded; the observer has therefore not to interrupt himself in order to write down the seconds and fractions; thus the wires may be much closer to one another and the whole observation take less time. The observation of transits of two stars which happen to be in the field together can even be carried on without additional difficulty and without confusion.*

57. The foregoing description of the transit instrument shews that to be in perfect adjustment it must satisfy the following conditions:

1st. The line of collimation must be perpendicular to the geometrical axis about which the instrument revolves.

2nd. This geometrical axis must be exactly horizontal.

3rd. It must point accurately east and west.

By satisfying the first condition the line of collimation describes a great circle; the second makes this great circle vertical, therefore passing through the zenith; and the third ensures its passing also through the pole and therefore coinciding with the meridian.

When these adjustments are not perfect, which is seldom, if ever, the case, there will be consequent errors called respectively *collimation*, *level*, and *deviation* errors. It is useless attempting to get rid of these errors altogether; they must be reduced as much as possible by mechanical means as described below, and the residual uncorrected errors must be carefully determined so that their effect on the observations may be calculated and allowed for.

* For a complete description of the process, see Loomis' *Practical Astronomy*.

Collimation Error (Mechanical Correction).

58. Place a graduated scale horizontally, at a long distance off, in such a position that the telescope may be directed upon it; and, when it is fixed there, note the reading intercepted by the middle wire. Then lift the telescope carefully from its bearings and replace it with the axis reversed, the left pivot being now in the right *Y* and the right pivot in the left *Y*.

Direct again to the scale, and if the reading shewn by the middle wire be the same as before, the adjustment is correct; if not, move the wires by the screws provided for the purpose, until the reading opposite the middle wire is the mean between the two previous ones. The error will now probably be corrected, but to verify it, repeat the operation by reversing the axis as before, and correct again if necessary until in both positions the same reading of the distant scale is obtained.

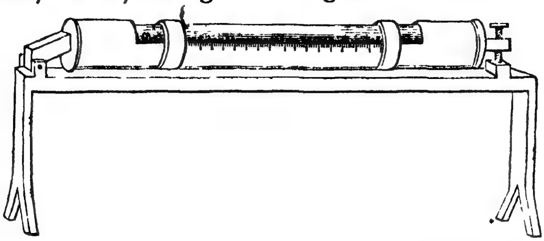
If the middle wire and the imaginary mean wire are not coincident there will still be collimation error; therefore all that must be attempted in the previous adjustment is as close an agreement as can be obtained without too much tampering with the screws. The collimation error of each of the wires, and thence of the mean wire, must be determined and allowed for afterwards. (See Arts. 74 and 80).

Level Error (Mechanical Correction).

59. To make the next correction we require a spirit level of sufficient length to reach from one extremity of the axis to the other, and furnished with legs terminating in inverted *Y*s to ride on the cylindrical pivots.

The tube of the spirit level is a small arc of a circular ring of very large radius placed with its convex side uppermost in a brass framework. It is nearly filled with spirits of wine or other quick fluid, a vacant space (called the bubble)

being left, which, owing to the gravitation of the liquid,



always occupies the highest part of the tube. A graduated scale attached to the tube allows us to ascertain the position of the bubble.

60. Place the level on the pivots, and read the graduations at each end of the bubble. Half the sum of these readings, supposing the zero to be at one end of the scale, will give the graduation at the middle. Now reverse the level, *i.e.* place the east end on the west pivot, and the west end on the east pivot; then, if the middle of the bubble settles to the same position as before, the axis is horizontal. If not, raise or depress that end of the axis which admits of vertical motion, until the middle of the bubble occupies the point midway between its two former positions. The axis will then be horizontal; but, to ensure accuracy, the operation should be repeated. (See Arts. 73 and 79).

Deviation Error (Mechanical Correction).

61. Supposing the errors of collimation and level to have been corrected, the line of collimation will describe a vertical circle, and the remaining error—that of deviation—will be detected as follows: Note the times marked by the clock when a circumpolar star passes the mean vertical wire, firstly above the pole, then, some twelve hours later, below the pole, and again above the pole after another such interval. If the two intervals are exactly equal the line of collimation describes a vertical plane which bisects the path of the star,

and therefore coincides with the meridian plane; but if not, it deviates from that direction, and the vertical circle which it traces out does not pass through the pole. The error is corrected by means of the screw which gives a horizontal motion to one end of the axis.

62. This method would not be available at places near the equator; the few stars which would be circumpolar stars there would, in their lower transit, pass too near the horizon to be observable. A method, which will apply in all cases, will be given below. (Art. 77).

Setting Circles.

63. A small graduated circle is attached vertically to one side of the telescope tube near the observer's end, and has a moveable diameter which carries a spirit level. This is called the *setting circle*, and its use is to bring the telescope rapidly to point to that part of the meridian where a star of known declination is about to cross. The moveable diameter is previously adjusted to the given declination on the graduated rim, then the motion of the transit about its axis, which is necessary to bring the bubble to the middle of the level, will just point the telescope to a star which has the given declination.

Sometimes there are two setting circles, one on each side of the telescope tube; and more, if thought necessary, might be so attached, but two are generally found sufficient. They are useful when it is requisite to take observations of two objects which follow one another rapidly, but differ considerably in declination.

Collimating Telescopes.

64. It was formerly considered essential in every observatory to have a distant *meridian mark* fixed due south of the transit, to verify its adjustments at any time.

The Observatory of Cambridge was built so that the transit instrument might be due north of the tower of Grantchester church, which happened to be in the required direction, and between two and three miles distant.

The necessity for a distant mark is now dispensed with by the following contrivance: The transit being perfectly adjusted, a small telescope, having cross wires at its focus, is fixed in a permanent position either to the north or south of the transit, at any convenient distance, and in such a position that, looking into its object glass through the transit telescope, we may see the cross (which must admit of being illuminated for the purpose) in exact coincidence with the middle wire of the transit. The rays from the cross wires will emerge from the small telescope in parallel directions, and, falling upon the object glass of the transit, will answer the purpose of a permanent mark at an infinite distance.

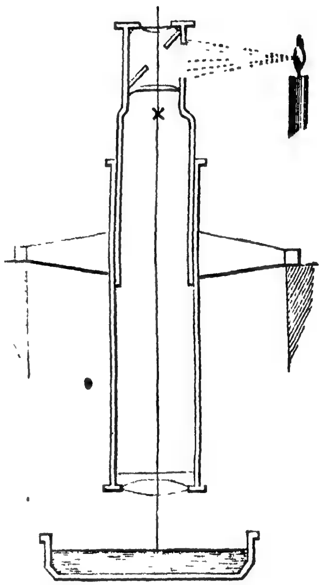
65. By using two such fixed telescopes, one on the north, the other on the south side, the adjustment for collimation may be made without the troublesome operation of reversing the transit, provided the two collimators be so placed that each may look into the other as well as into the transit. To ensure their doing so, without removing the transit from its supports, it must be turned to point to the zenith, and through two openings or doors managed in the sides of the tube, the one collimator may be adjusted on the other.

Then if the transit, after being adjusted on the south collimator, be turned towards the north one and found to be in adjustment with it also, the line of collimation will be at right angles to the axis of rotation.

Collimating Eye-piece.

66. When the level error has been corrected, the collimation adjustment may be very simply and accurately made, by pointing the telescope vertically downwards to a vessel of mercury placed below it. The rays from any point of

the wires will proceed in parallel directions from the object glass, and, after reflexion at the horizontal surface of the mercury, will return to the object glass in parallel directions, making with the vertical, on the other side of it, the same angle as before. An inverted image of the system of wires is thus formed at the focus, and when the middle of the mean wire coincides with its image, there is no collimation error. (See Art. 74).



In order to see the wires, an eye-piece is used called the collimating eye-piece, which has a lateral opening between the two lenses. Through this opening, light is introduced and reflected towards the wires, whose dark side being towards the mercury, the real wires will be seen bright, and their reflection dark. The light is reflected towards the wires by means of an elliptic ring reflector, or by a piece of plate glass which, without interfering with the direct vision, reflects sufficient light to render the wires visible.

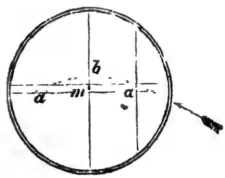
Reduction of Observations to the Mean Wire.

67. We have said that five or seven vertical wires are used, and the mean of the times taken for the time of transit. A passing cloud or other accidental circumstance may cause us to miss the star at the moment of its passage across some of the wires, but the observation will not be lost if the transit across one or more of the wires is secured, provided the time be known which the star would take to pass from each wire to the imaginary mean.

This interval will not be the same for all stars, those near the equator moving more rapidly than those near the pole; but there will be a simple connection between them, so that when the equatorial intervals are known, those for a star of given declination will be readily obtained and may be tabulated for use.

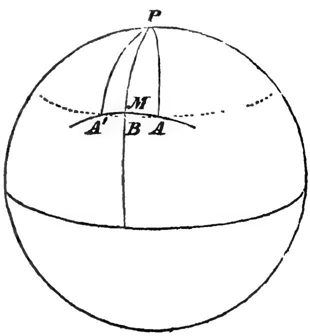
68. A star, its image, and the centre of the object glass, are always in a straight line; hence, as a star moves, the straight line joining it with the centre of the object glass traces out, by its intersection with the plane of the wires, the path of the image across the field of view. Now, considering the centre of the object glass as the vertex of the cones in fig., p. 3, we see that for a star in the equator this path will be a straight line, but for all others it will be an arc of a circle having its concavity towards or from the north pole, according as the declination of the star is north or south.

The telescope being an inverting one, let a, b, a', represent the apparent path of a star crossing the meridian between the zenith and the equator. The arrow indicates the direction of motion in north latitudes. Let a be the point of intersection of the horizontal ama' with the wire whose interval is required, m being the mean wire.



Let A, M, A' be points of the celestial sphere corresponding to a, m, a'.

AMA' will be on a great circle perpendicular to the meridian PM, and the interval required is the same fraction of 24 hours that the angle APM is of 360°.



The right-angled triangle APM gives

$$\sin AM = \sin APM \sin PA\,;$$

but AM depending on am is constant for all positions of the telescope, and for a star in the equator $\sin PA = 1$; therefore AM is the equatorial value of APM, that is AM is the equatorial interval of that wire. Call it w, then

$$\sin w = \sin APM \cos \delta$$

will give the value of APM when w is known.

When the star is on, or within a couple of degrees of the equator, $\cos \delta$ will be very nearly 1, and $APM = w$.

Except for stars near the pole, w and APM will be both small, so that

$$w = APM \cos \delta \text{ nearly.}$$

69. To determine w: Observe the times of transit of a star of known declination δ across each of the wires, and take the mean. From this mean subtract the time of crossing the first wire, this will give the angle APM corresponding to that wire, whence, by the above formula, the value of w may be calculated. In the same manner determine $w_2, w_3, \ldots w_7$, for the second, third, &c., wires. Those which precede the mean are positive, the others negative, and their sum is obviously zero.

Delambre (*Ast.* vol. I., p. 416)* seemed to give the preference to stars on, or near, the equator; but, in order that an error in the observation of APM may have the less influence on the value of w, it is usual to select a star near the pole; for, although the probable error in the observed value of APM increases with the declination, on account

* Toutes les étoiles peuvent ainsi servir à déterminer l'intervalle équatorial; le plus court est d'y employer les étoiles dans l'équateur, qui n'exigent aucun calcul. Un ou deux degrés de déclinaison n'apportent aucune différence sensible. Quelques Astronomes ont cru qu'il y avait de l'avantage à choisir les étoiles circompolaires, parce qu'elles se meuvent beaucoup plus lentement, mais l'avantage que procure cette lenteur est plus que détruit par l'incertitude de l'observation.

of the more oblique and more sluggish motion of the star across the wire; yet, except for stars quite close to the pole, this increase will be more than counterbalanced by the advantage of a decrease in the cosine of the declination.

70. The equatorial intervals of the wires having been determined as above from the mean of a large number of observations on different stars, we shall proceed to shew how they may be applied to the correction of an imperfect observation. Suppose a star observed at the second, fifth, sixth, and seventh wires at the times t_2, t_5, t_6, t_7. The estimated time across the mean wire will, by adding the corresponding *APM* to each, be

$t_2 + w_2 \sec\delta$ from the first wire observed,

$t_5 + w_5 \sec\delta$ .... second ....,

$t_6 + w_6 \sec\delta$ .... third ....,

$t_7 + w_7 \sec\delta$ .... fourth ....,

adding and dividing by the number of wires, we obtain for the time of transit

$$\frac{t_2 + t_5 + t_6 + t_7}{4} + \frac{w_2 + w_5 + w_6 + w_7}{4} \sec\delta,$$

i.e. multiply the algebraical mean of the equatorial intervals, corresponding to the wires observed, by the secant of the declination; the product will be the additive correction to be applied to the mean of the times.

Determination and Effect of Residual Errors.

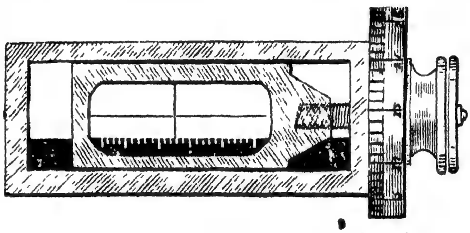
71. The errors of collimation, level, and deviation, having been almost completely corrected by some of the mechanical methods indicated in the preceding pages, we will now proceed to indicate the means of determining and allowing for the small residual errors; but as the angles we shall have to consider are extremely small, we shall first explain the

means employed for measuring such small angles with great accuracy.

Spider Line Micrometer.

72. The instrument of greatest value for this purpose is the spider line micrometer. It consists of a small rectangular frame-work, about three or four inches in length, by one inch in breadth, carrying a wire or spider line, which can be moved by means of a screw. The frame-work is placed at the common focus of the object glass and eye-piece of the telescope so that the spider line may be almost in contact with the fixed wires.

The threads of the screw should be perfectly uniform and



regular, so that each turn of the screw-head may carry the spider line over an equal space.

The value of one turn of the screw, *i.e.* the angle, subtended at the centre of the object glass, by the distance through which the line is shifted, can be measured by the method explained for determining the equatorial intervals of the wires (Art. 69). Make the spider line coincide with one of the wires, then separate them by one or more turns of the screw-head, and divide the corresponding equatorial interval by the number of turns.

In this way also may the regularity of the screw be tested throughout, the head being divided into sixty or one hundred parts to indicate the advance of the thread corresponding to the sixtieth or hundredth part of a revolution.

In order to check the number of turns, a row of teeth or notches, called a comb, is visible at one side of the field

of view. The line passes one of these at each revolution of the screw-head, and every fifth one is cut deeper than the others. The central one of all is distinguished by a small circular aperture.*

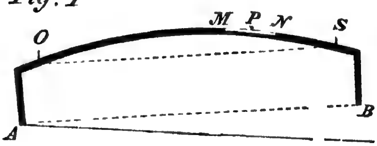
Determination of Uncorrected Level Error.

73. In the description of the spirit level (Art. 59) we said that a graduated scale is attached to the tube. If we raise or lower one end of the tube, the bubble will move along the scale, and the change of inclination corresponding to each division may be easily determined experimentally, and expressed in parts of a second.†

If we place the level on the pivots and take the reading of each end of the bubble, then half the algebraic sum of the readings will be that of the highest point.

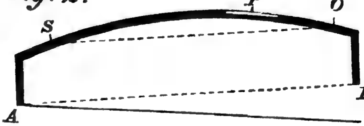
Let A, B (fig. 1) be the two pivots, MN the bubble, P its middle point, O the zero of the scale, S the point corresponding to O at the other end of the scale (so that OS is parallel to AB).

Fig. 1



If A and B were on the same level, P would occupy the middle point of OS. The inclination will therefore be measured by the difference between OP and $\frac{1}{2}OS$.

Fig. 2.



To determine OS, turn the level so that the east end may be over the west pivot, and west end over the east

* Sometimes two parallel wires are used, each moveable by its own screw. For a description of the method of using these, see Chauvenet's *Astronomy*, and Loomis' *Practical Astronomy*.

† The simplest way of doing this, is by attaching the level to a mural circle. The angle through which the level turns, as the bubble passes from one graduation to the next, is at once given by the mural.

pivot, as in fig. 2; S and O interchange places, and the middle of the bubble being at P', the mean reading will now give OP', which is equal to SP of fig. 1.

Let OP, or a be the reading of P in the first position,
OP', or b .... P' ......second.........;
therefore $OS = a + b$; and the inclination of AB to the horizontal, that is the level error, is $\alpha(OP - \frac{1}{2}OS)$, that is $\frac{1}{2}\alpha(a - b)$, where α is the value in seconds of each division of the scale.*

Determination of Uncorrected Collimation Error.

74. Apply the collimating eye-piece and the micrometer to the transit. Then, turning the telescope vertically downwards to a trough of mercury, as described in Art. 66, measure the angular distance between each wire and its image. One half of this angle will be the inclination to the vertical of the line which joins the wire with the centre of the object glass.

Correct this for any level error there may be, and the result will be the collimation error of the wire in question; that is, the angle which the line joining the wire with the centre of the object glass makes with the line at right angles to the axis of rotation.

Correct this again for the interval in seconds of arc between this wire and the mean of the seven (Art. 69). This will give a value of the collimation error of the mean wire.

Repeat the operation on each wire and take the mean of the results for the collimation error of the transit.

75. If the level error be not allowed for, the preceding result will be the *sum* or *difference* of collimation and level errors. But if the transit be lifted off its Ys and reversed,

* The figure supposes O, the zero of the scale, to be near one end of the instrument, and therefore S near the other. If O were at, or near. the middle, and O and S on the same side of P. OS would be $a - b$ and the level error $\frac{1}{2}\alpha(a + b)$.

The triangle ZPM gives

$$\cot PM \sin PZ = \cot Z \sin P + \cos PZ \cos P,$$

$$\cot \Delta \cos \phi = - \cot x \sin P + \sin \phi \cos P,$$

but x and P are both small; therefore

$$P = x \{\sin \phi - \cot \Delta \cos \phi\} \quad \text{.............. (1).}$$

So the triangle ZPN gives

$$P - \theta = x \{\sin \phi - \cot \Delta' \cos \phi\};$$

therefore

$$\theta = x \{\cot \Delta' - \cot \Delta\} \cos \phi,$$

$$x = \frac{\theta \sin \Delta \sin \Delta'}{\cos \phi \sin (\Delta - \Delta')}.$$

78. In selecting stars for observation by this method, which is always applicable, it will be desirable to take two whose right ascensions are not widely different; because the time between the two transits being short, the chance of error in the observed value of θ, so far as it is due to any irregularity of the clock, will be diminished. Again, the larger θ is, the less will an error in its value affect the value of x; but θ varies as $\cot \Delta' - \cot \Delta$, therefore the two stars should differ considerably in declination.

Corrections to be Applied to the Observed Time of Transit.

79. Let u, v, x be the values of the uncorrected errors of level, collimation, and deviation respectively, determined by some of the preceding methods. We will now calculate their effects on the time of transit of a star; and in doing this, we may suppose them to exist separately, the aggregate correction being the sum of the partial corrections when the errors are small.

Effect of Uncorrected Level Error.

Let RZP be the meridian, and

RAH the great circle traced out by the transit,

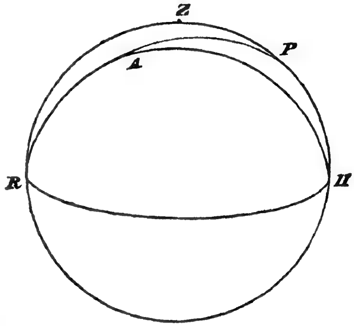
u the level error or angle at R between the two circles,

Δ the polar distance of a star seen in the transit at A,

ϕ the latitude $= 90° - PZ$,

θ_1 the error APZ in the time of transit.

The triangle PAR gives



$$\cot PA \sin PR$$

$$= \cot R \sin P + \cos PR \cos P,$$

$$\text{or} \quad \cot\Delta \sin\phi = \frac{\theta_1}{u} - \cos\phi,$$

$$\theta_1 = u \frac{\sin(\Delta+\phi)}{\sin\Delta}.$$

This being the correction to be added to the observed time of transit, we see that u must be considered positive when the western pivot is too high, and negative when too low. For a transit below the pole the reverse would be the case.

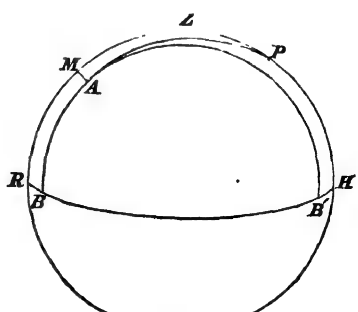
Effect of Uncorrected Collimation Error.

80. Let the transit trace out the small circle BAB' parallel to the meridian $RMPH$, and let AM be a perpendicular from the star A to the meridian. Join AP.

Let $v = AM$, the collimation error,

$\theta_2 = APZ$, the error in the time,

$= \sin AP \sin APM$,



$$\theta_2 = \frac{v}{\sin\Delta}.$$

v is to be considered positive when the line of collimation points to the east of the meridian, and negative when it points to the west.

Effect of Uncorrected Deviation Error.

81. As in Art. 77, the triangle ZPM will give the equation

$$\cot PM \sin PZ = \cot Z \sin P + \cos PZ \cos P,$$

whence $$P = x(\sin\phi - \cot\Delta \cos\phi),$$

or $$\theta_3 = -\frac{x\cos(\Delta+\phi)}{\sin\Delta}.$$

The fig. of Art. 77 shews that x is to be reckoned positive when the east pivot of the axis of the transit deviates towards the elevated pole, and negative when it deviates in the other direction.

82. For the complete correction, therefore,

$$\theta_1 + \theta_2 + \theta_3 = \frac{u\sin(\Delta+\phi) + v - x\cos(\Delta+\phi)}{\sin\Delta},$$

where u, v, and x are to be taken with their proper signs, as explained above; and the result being expressed in seconds of arc will have to be divided by 15 to reduce it to seconds of time.

Personal Equation and other Errors.

83. Besides the errors we have spoken of, which may be called *errors of adjustment*, there are several other sources of error to which all instrumental observations are liable.

Firstly, *errors of construction* arising from some imperfection of workmanship. These can only be detected by a thorough study of each individual instrument, and by making repeated observations under varied circumstances; but a knowledge of the theory of the instrument will often suggest modes of eliminating these errors; as, for instance, in the case of the mural (Art. 93), where the using opposite microscopes in pairs corrects a possible error in the position of the centre of graduation.

Secondly, *accidental errors* due to extraneous causes, such as sudden and unobserved changes of temperature, atmo-

spheric indistinctness, looseness of screws, &c. The observer has no means of correcting these errors, except by multiplying his observations, whereby he may hope that, as the errors are accidental, they will sometimes tend one way and sometimes the other, and therefore that the mean result will have a less error than any single observation.

Thirdly, a kind of error to which the name of *personal error*, or personal equation, has been given. It is often found that two observers, though equally trained in observing, will differ by a fraction of a second in their estimation of the time of transit of a star, and that this difference remains pretty nearly constant for months or years.* One of them is too slow or the other too precipitate, but at any rate a discordance exists which, though small, must not be neglected when we have to combine observations made by the two. In the same observatory it is usual to select some one of the observers as a standard of reference, and to reduce all observations made by the others to the standard, by allowing for their personal equation as obtained by comparison with him.

* See Arago's *Mémoires Scientifiques*, tome II., p. 233.

CHAPTER V.

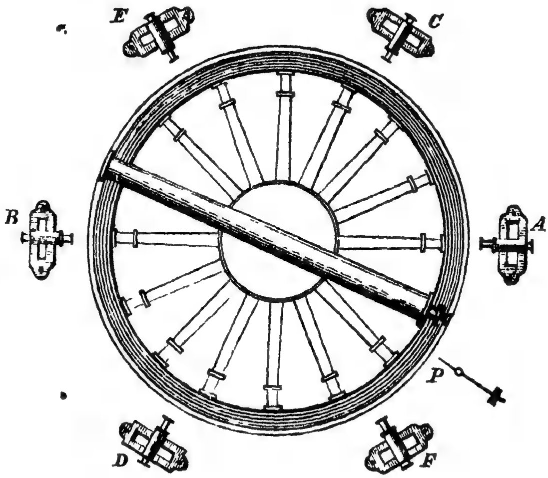
THE OBSERVATORY CONTINUED.

The Mural Circle.

84. THE mural circle determines the meridian zenith distance of an object, and is a necessary complement of the transit. It consists of a large graduated circle placed in a vertical plane against the eastern face of a wall which runs north and south. This circle revolves in its own plane about a strong axis which penetrates right through the wall, and which, for purposes of adjustment, can be slightly moved by screws on the western face. An astronomical telescope is firmly attached to the face of the circle, so as to be parallel to a diameter, and when the whole instrument is in perfect adjustment the line of collimation (Art. 86) describes the meridian. The graduations of the circle are made on the thickness of the rim, and read continuously from 0° to 360° at intervals of 5′. These divisions pass successively behind a small projecting piece of metal attached to the wall, called the *pointer;* and the pointer-reading, when the telescope is turned exactly to the zenith, is called the *zenith-point.* The difference between the zenith-point and the pointer-reading, when the telescope is directed towards some star, is the meridional zenith distance of that star.

85. The graduations are, as we have said, at intervals of 5′. The intermediate minutes and seconds are obtained by means of six microscopes firmly fastened to the wall, placed round the circle 60° apart and looking towards the

graduated rim. One microscope would be sufficient, but six are used to counteract errors of graduation, centering, and unequal expansion (Art. 93). They are lettered and



referred to as in the figure, *A*, *B*, *C*, *D*, *E*, *F*. Each microscope is furnished with a micrometer, similar to the spider-line micrometer described in Art. 72. It is convenient, though not essential, that the microscopes be so adjusted that, when the pointer coincides with a graduation, all the micrometer wires, supposing each to occupy its zero position at the middle of the field of view, should also coincide with graduations. If the circle be then turned through a fraction of a division, an equal portion of the limb will pass opposite to each microscope:—the precise fraction of arc can be obtained with extreme accuracy, by the number of turns and parts of a turn of the screw-head, necessary to carry the spider-line from the zero position till it coincides with the image of the division that has passed away from it.

Five turns of the screw-head should carry the spider-lines from one division of the limb to the next, so that each turn may indicate one minute; a notched comb of metal at the side of the field will check the number of turns or minutes. The screw-head being divided into sixty equal parts, the reading off will give the additional seconds, and the mean of the six readings, added to the integral number of 5′ shewn by the pointer, will give the reading of the mural for that position.

86. At the principal focus of the telescope is a framework carrying one horizontal and five vertical wires;—the line of collimation is the line joining the centre of the object glass, with the intersection of the horizontal wire by the mean of the five.

All these wires are fixed; but another horizontal wire, called the micrometer wire, moveable across the field of view, is necessary in certain observations for measuring small distances of celestial objects from the fixed horizontal wire. These measurements are made in minutes and seconds of arc,* by means of a graduated screw-head, which gives motion to the micrometer wire, and whose graduations have a known value. The minutes and seconds so obtained, added to, or subtracted from, the mural reading given by the pointer and microscopes, will give what the mural reading would have been, if the fixed wire had occupied the position of the micrometer wire.

The values of the graduations for the micrometer wire may be found thus:—Direct the telescope towards some distant mark which happens to be, or is purposely placed, in the meridian of the mural. Move the telescope micrometer, by giving the screw-head some exact number of revolutions from its zero position, say ten for instance. Then turn the mural gently till the micrometer wire just bisects the mark, and read off the pointer, and the six

microscopes. Repeat the operation with the micrometer wire placed at ten revolutions on the other side of the zero, and again read the circle. The difference of the two readings will be the value, in arc, of twenty revolutions of the screw. Hence the value of one revolution, and of its fractions, will be known.

Clamp, Tangent Screw.

87. In making an observation with the mural—when it has been turned so that the star is in the field of view, and a very small additional motion is required to bring it to the fixed wire—it will generally be found that the hand is too abrupt in its movements, and that a slower and more delicate means is necessary. This is obtained by a clamp and tangent-screw, of which there are several, so that, in all positions, one may be within reach of the observer.

The *clamp* consists of two lips of metal between which the rim of the circle slides. These lips project from another piece attached to the wall, and by one turn of a screw they can be tightened on the circle so as to fix it. The *tangent-screw* is another screw, with very fine threads, which, acting on the piece that carries the clamp, moves it and the circle with it at a slow pace in the direction of the arc, thus modifying the too rapid motion of the hand.

Error of Runs.

88. When a change of temperature, or any other cause, alters the distance of a microscope from the rim of the circle, the distance, at the focus, between the images of two consecutive graduations, will alter; and five turns of the screw-head will no longer carry the micrometer wire accurately from one division to the next. The error arising from this is called the error of runs, and a corresponding correction becomes necessary.

Suppose the interval between two graduations at the microscope A, to require a'' more than the five turns, the correction to be applied to a smaller interval, measured by the microscope, must be obtained from the proportion,

$$5' + a'' : a'' :: \text{measured interval} : \text{correction}.$$

This correction must be subtracted when a is positive and added when a is negative.

In the same way may the reading of each microscope be corrected; but instead of applying the correction to each, which would necessitate six proportions, it will be sufficient to correct the average measured interval by means of the average error; and as a is always a small number of seconds, we may further simplify the proportion by using $5'$ for the first term instead of $5' + a''$.

The errors of runs must be examined from time to time, and if they become too considerable, they may be mechanically reduced by moving the microscopes towards or from the circle.

Errors of Adjustment of Telescope.

89. The line of collimation of the telescope should describe the meridian:—the instrument is therefore, like the transit, liable to the three errors of collimation, level, and deviation. But these errors have not the same importance as in the case of the transit; for, the object of the mural being the determination of altitudes, a very small deviation from the meridian will not produce an appreciable error in the result; and it will generally be sufficient to correct, as far as possible, by mechanical adjustment.

90. Level error may be detected by means of a microscope, projecting from the face of the mural, with its axis parallel to the arc of the rim and revolving with it. Let a fine plumb line, fastened to some point above, pass in front of the circle just opposite its vertical diameter, and at such

a distance from it that it may be viewed by the microscope when near its highest position. Then turn the mural half round, so as to bring the microscope to its lowest position, and the thread should again occupy the same position in the field of view. If it does not, we must correct the error by means of the adjusting screws at the back of the wall.

91. Supposing the level error corrected, we may proceed to correct the collimation error, if there be any. This will be done by moving the framework, which carries the wires of the telescope, so that a star near the zenith may pass the middle wire at the same moment that it passes the middle wire of the transit.

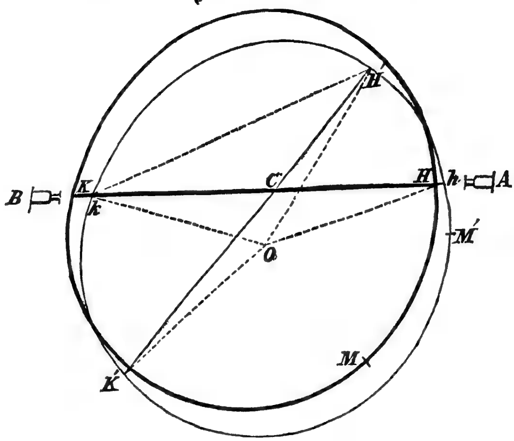
92. The deviation error is next corrected, by observing a star near the horizon, and moving the axis by means of the screws ,at the back of the wall, so that the star may cross the middle wire of the mural and that of the transit at the same instant.

Errors of Centering, Graduation, &c.

93. The mural is essentially a differential instrument, requiring always two positions to complete an observation. The angle through which the telescope turns is the quantity we have to determine, and we can easily shew that by using reading microscopes in pairs, we may eliminate any error which arises from the centre of graduation not coinciding with the centre of rotation, or from irregularity in the form of the pivots.

Thus, let the dark line in the figure represent the first position of the graduated circle when H, K are the points of the limb opposite the microscopes A, B, and M the zero of graduation. Let the fine line represent the second position of the circle, O its centre, the line HK having come into the position $H'K'$, and M moved to M'. The telescope will have

turned through the same angle as any line rigidly connected



with the circle; therefore HCH' is the angle to be found.

The two readings of

A are respectively (MH or) $M'H'$ and $M'h$,

B .... (MHK or) $M'H'K'$ and $M'H'k$.

The difference of the readings at A gives $hOH' = 2\,CkH'$,

.... B ...... $kOK' = 2\,CH'k$.

The sum of these $= 2\{CkH' + CH'k\}$

$= 2HCH'$,

and the half sum is the angle between the two positions of the telescope.

The centre of rotation, that is the point about which the instrument is supposed to have turned in passing from the first position to the second, is not shewn on the figure; and the investigation, being independent of this centre, shews that, even when the pivots are not cylindrical, and when consequently the centre of rotation is not a fixed point, the readings of each pair of microscopes will give a value of the angle free from errors of centering, and from irregularity in the form of the pivots. The mean of the three pairs will diminish the risk of errors of graduation and of unequal expansion.

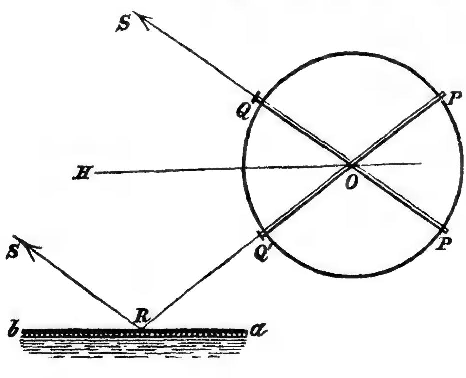
Observing with the Mural. Zenith-Point. Polar-Point.

94. The difference of declination of two stars, or, which is the same thing, the difference of their meridional zenith distances, may be obtained as follows: When the first star is about to pass the meridian, turn the telescope so that it may be in the field of view. Then clamp the mural, and, by means of the tangent screw, move the circle, so that the star may be on the fixed horizontal wire* at the moment that it crosses the middle vertical one. Now read the six microscopes, and add the mean of the six readings to the pointer reading.

Repeat the operation on the second star and take the difference of the results, this will give the difference of meridian altitudes of the two stars.†

95. If, instead of a second star, we use the image of the same star as seen by reflection the next evening in a trough of mercury, we shall obtain the meridian altitude of that star.

Thus, if PQ be the direction of the telescope on the first night, when we observe the star S directly, and $P'Q'$ its



direction on the second night when we observe it by reflection

* Some instruments, instead of a fixed horizontal wire, have two fixed wires separated by an interval of about 10″, and the star is kept exactly in the middle of the space between the wires.

† These results will require correction for refraction. See Chap. XV.

at R, the surface of the mercury, then QOQ' is the angle given by the difference of the two mural readings.

Now OS is parallel to RS, and if OH be drawn horizontal, and consequently parallel to the surface aRb of the mercury,* the angles SRb and ORa being equal, QOH and HOQ' will be equal, hence $SOH = \frac{1}{2} QOQ'$; or, *half the difference of the mural readings will give the altitude of the star.*

96. It is also obvious, that half the sum of the readings will be the reading which the instrument would give if the telescope coincided with the horizontal OH. This horizontal reading, increased or diminished by 90°, as the case may require, gives the zenith reading or *zenith-point.*

When the zenith-point is known, one observation of a star is sufficient to determine its altitude:—Taking the difference between the zenith-point and the mural reading corresponding to the direct observation of the star, we obtain its meridional zenith distance.

97. One objection to the preceding method of observation and of determination of the zenith-point is that, in the interval between the observations, changes of temperature, &c., may occur, altering the amount of refraction of the star, and therefore its apparent direction; and there is also this inconvenience, that, in consequence of unfavourable weather, several days may intervene before a complete observation can be obtained.

The following method, known as "a double observation with the mural," has been devised for observing both by direct and reflected vision at one and the same transit.

Some time before the star (whose zenith distance is supposed approximately known) comes to the meridian, set the circle by the pointer, so that the image reflected from

* If the mercury were at a considerable distance from the mural, it might become necessary to take account of the angle between the normals at O and R.

the mercury may be sure to enter the field of view. Then clamp the circle and read the pointer and the six microscopes. When the star enters the field of the telescope, follow it with the *micrometer wire* so as to bisect it just before it crosses the middle vertical wire. Unclamp the circle and turn it till the star, seen directly, is again in the field of view (which can be rapidly done by the approximately known pointer reading). Then, using the nearest clamp and its tangent screw, bisect the star by the *fixed horizontal wire.* Finally, read off the pointer, the six microscopes, and the micrometer of the telescope.

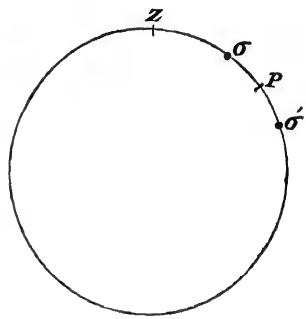
The former reading of the mural corrected by adding, with its proper sign, the micrometer reading will reduce the reflection observation to the fixed horizontal wire; and we shall thus have two circle-readings corresponding to direct and reflection observations of the same star.

Half the sum of these readings will give the horizontal point and half the difference the meridian altitude of the star.

98. We here suppose that the star is so near the meridian at the two moments when it is observed that the error of altitude is inappreciable. But, if the interval were considerable, for instance, if the first observation were made at the moment when the star is crossing the first vertical wire, and the second when it is crossing the last, it would be necessary to calculate the change of altitude to the meridian. The formula for this correction is given in the method of finding the latitude by circum-meridian observations. See Chap. IX.

99. The zenith-point may also be obtained independently and very accurately by means of the collimating eye-piece. We may, as was described in the case of the transit (Art. 66), make the line of collimation exactly vertical by pointing the telescope vertically downwards towards a trough of mercury, and bringing the reflected image of the fixed horizontal

wire to coincide with the wire itself. The corresponding mural reading will be the nadir-point, which, increased by 180°, will give the zenith-point.



100. *To find the Polar-point, i.e.* the reading of the mural when the telescope is directed to the pole.

Observe a circumpolar star in its upper and lower transits, and note the readings of the mural in the two positions.

Thus, if

$Z\sigma P\sigma'$ be the meridian of the observer,

σ the star at the upper culmination,

σ' .... lower ....,

H the polar-point and Δ the polar distance,

then $P\sigma = P\sigma' = \Delta$,

and one of the readings will be $H + \Delta$, the other $H - \Delta$.*

Therefore the polar-point is given by half the sum of the two readings.

COR. 1. Half the difference of the two readings is the polar distance of the star, which subtracted from 90° gives the declination.

COR. 2. The difference between the polar-point and the zenith-point is the colatitude ZP, which, subtracted from 90°, gives the latitude.

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* These readings must be corrected for refraction and for instrumental errors.

CHAPTER VI.

THE OBSERVATORY CONTINUED.

The Transit-Circle.

101. ALTHOUGH the transit and the mural have been thus described as the two important instruments in an observatory, it is now found that a single instrument, combining the functions of both, can with advantage be made to take their place. Theoretically, the mural would do so, if the adjustment in the meridian could be relied on; but its peculiar mounting, requiring the support to be all on one side, makes it unable to bear the weight of a large instrument, and renders it liable to deflections from the meridian—deflections which are unimportant in their effect on the altitudes, but which could not be tolerated in the determination of the times of transit.

A *Transit-circle* has been recently mounted at the Cambridge Observatory in the place of the former Transit instrument. It is symmetrically made and supported. The axis, resting on the two piers, carries, besides the telescope, two graduated circles, one on each side, about three feet in diameter, and revolving with the instrument. Two other circles, firmly fixed to the piers, carry the reading-microscopes, four on each, besides a pointer-microscope, which replaces with advantage the metal pointer of the mural.

The telescope has a focal length of about nine and a-half feet and an aperture of eight inches. The instrument is reversible, but there is little need for this provision, because two collimating telescopes, with apertures of six inches, have

been fixed on the north and south sides, and the adjustments can be accurately made without reversion (Art. 65).

The counterpoises are not seen, being under the floor, where they act on levers that thrust upwards and support the weight. The graduations opposite the reading-microscopes receive light by an ingenious system of glass prisms; and various other devices, too technical to be mentioned here, combine to render this a most effective and valuable instrument.

One observation with a transit-circle gives the two results which, with the transit and mural, required two observers. There is thus a great saving of labour; and one source of error, from which the separate instruments were not free when observing small stars thickly clustered, is altogether impossible in the transit-circle:—the error, namely, of observing one star for altitude and, by mistake, a different one for time, so as to create an imaginary star combining the elements of two real stars.

The adjustments and corrections for level, collimation, and deviation are made as for the transit, and need not therefore be dwelt upon.

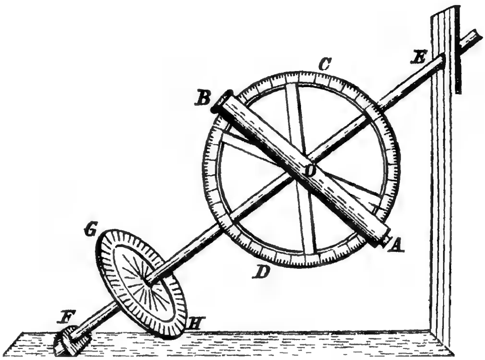
The Equatorial.

102. The only observations which can be made with the instruments, so far described, are restricted to the plane of the meridian; but there are many occasions, such as eclipses and occultations, where the phenomena to be observed occur out of that plane.

It may also be desirable to have a means of fixing with accuracy the position of an object at any moment when visible, especially of those objects which, like comets, have a rapid motion among the others. We therefore require telescopes so mounted as to allow of being pointed in any direction, and an especial advantage is secured when the object can be followed with ease, and retained in the field

of view, so as to afford sufficient time for micrometer measurements and minute examination.

103. One of the instruments devised for the purpose consists of a telescope AB in contact with a graduated circle



CD, and turning about an axis through the centre O perpendicular to the plane of the circle. One of the diameters of this circle is produced, and constitutes an axis EF, whose direction coincides with the axis of the celestial sphere. About this axis the whole instrument turns, and the angles through which it turns are measured by a second graduated circle GH, perpendicular to the axis.

104. If the telescope be directed towards a star, EOB will be the angular distance of that star from the pole; the polar distance, or the declination, may therefore be measured by the graduated circle CD, and, by merely turning the plane of this declination circle, without altering the angle EOB, we may follow the star during its diurnal course, and thus verify the accuracy of the statement which the rough observations in Art. 5 suggested, viz. that the stars move uniformly in circles round the axis of the celestial sphere.*

* Allowance must be made for refraction, as will be explained hereafter.

In most instruments the circle *GH* is connected with clock-work, the motive power being a descending weight and the regulator a conical pendulum or governor. A uniform motion without jerks is thus communicated to the instrument, which, when properly adjusted, can retain in the field of view a star or other object, so long as may be required, without any attention on the part of the observer.

But the principal use of the instrument is for differential observations:—to find the difference of declination or of right ascension of two neighbouring bodies, the diameters of planets, the distances of double stars, &c. These observations can all be made with the instrument in a fixed position, and are therefore independent of errors of adjustment.

105. The above is the principle of all equatorials. The mounting may be different in different instruments, but in every one there must be—a fixed line coincident with the polar axis, a means of measuring the angle which the telescope makes with this fixed axis, and also of measuring the angle described by the instrument in turning about the axis.

There will therefore be six adjustments:

(1) The polar axis must have the same altitude as that of the celestial sphere.

(2) The declination circle must mark *zero* when the line of collimation is in the plane of the equator.

(3) The polar axis must be in the plane of the meridian.

(4) The line of collimation of the telescope must be at right angles to the declination axis (the axis through O about which the telescope turns).

(5) The declination axis must be perpendicular to the polar axis.

(6) The graduated circle *GH* must mark *zero* when the telescope is in the meridian.

For the means of performing these adjustments we shall refer to works on practical astronomy. Loomis' *Introduction to Practical Astronomy*, Challis' *Syllabus*, &c.

106. The necessity for these corrections, and the liability of a heavy instrument mounted in this inclined position to get out of adjustment, render the equatorial somewhat uncertain in the determination of absolute declinations and right ascensions; but the facility of pointing it in any direction makes it a valuable means of determining the differences of right ascension or of declination between objects which both pass through the field of view while the telescope is in a fixed position.

There are, as we have stated, various ways of mounting the equatorial, but the differences are points of detail which can easily be understood: for instance, in the Northumberland equatorial at the Cambridge Observatory, there is no declination circle, but a straight rod—which, by sliding in a tube, can be shortened or lengthened at pleasure—connects the eye-end of the telescope with a point in the axis of the instrument, and thus determines the angle at which the telescope is inclined to the polar axis. The angle itself is shewn by graduations on the rod.

107. At the focus of the object-glass of the equatorial is a system of spider lines or wires, similar to those of the transit, and respectively parallel and perpendicular to the plane of the declination circle. Besides these, there must be moveable wires for micrometer measurements, such as those of the spider line micrometer, already described in Art. 72.

Position Micrometer. It is often necessary when two objects are in the field of view at the same time, to find the angle which the line joining them makes with the declination circle passing through either of them and the pole. For this purpose the eye-piece is furnished with a

spider-line which, by means of a rack and screw, admits of circular motion in a plane at right angles to the line of collimation. If the telescope be pointed towards one of the objects and the spider line be turned to cover them both, then the angle it makes with the fixed central wire is the angle required, and will be given by the screw-head.

108. *Finder of Equatorial.* When the equatorial is a large instrument of great magnifying power, the portion of the heavens embraced by the field of view becomes so small, that it is somewhat difficult to point the telescope. A smaller telescope called the *finder*, whose line of collimation is parallel to that of the equatorial, is usually attached to the larger instruments: this embraces a much wider field, and a star brought to the centre of the finder will be in the field of the equatorial.

Altitude-Azimuth—Transit in the Prime Vertical.

109. The altitude and azimuth instrument, or alt-azimuth, is another instrument intended, like the equatorial, to make observations on objects situated out of the meridian plane. As its name implies, it gives both the altitude and the azimuth of a celestial body. It consists of two graduated circles—the one fixed in a horizontal plane, the other in a vertical plane which admits of being turned about a vertical axis so as to coincide with any azimuth. A telescope, parallel to the plane of this vertical circle, turns with it in azimuth and admits also of motion in altitude.

It is, in fact, an equatorial with the axis pointing to the zenith, instead of to the pole; and if we hold the figure, p. 81, so that *FE* may be vertical, we shall have an alt-azimuth. The circle *CD* will measure the altitude, and the circle *GH*, the azimuth of any body towards which the telescope is pointed.

An alt-azimuth has been in use at Greenwich since 1847, with the special object of observing the moon off the meridian.

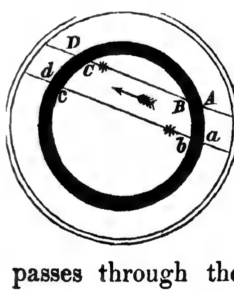
A detailed description will be found in the *Greenwich Observations* for that year.

By clamping the horizontal circle, the alt-azimuth telescope can only describe a vertical circle. This vertical circle may coincide with the meridian, in which case it becomes a transit instrument; or, it may coincide with the prime vertical, and, in this position, observations of such peculiar value can be made that a special instrument called a *Transit in the Prime Vertical* has, for some years, been in use in the Pulkowa Observatory. For a description of it we shall refer to Chauvenet's *Astronomy*.

The Ring-Micrometer.

110. All the instruments so far described require to be mounted in a special manner for the purpose of observing; but very good observations of *differences* of declination and right ascension can be made in a very simple manner with an ordinary telescope fitted with the contrivance known as a ring-micrometer. This consists merely of a flat ring of metal with very fine edges, which is made perfectly circular and fixed at the focus of the telescope.

When the difference of declination of two stars is so small that both may pass through the field of the telescope in a fixed position, let the telescope be so pointed that these passages may take place, and note the times when one of the stars passes behind the ring at A and C, and reappears at B and D. The mean of these times will be the instant when it passes the middle point of the chord; that is, it will be the instant when the star is in the declination circle which passes through the centre of the ring, for the path of a star will obviously cut every declination circle at right angles.



The same being done for the other star, care being taken not to disturb the telescope, the difference of these two means will be the difference of right ascension of the two stars.

To determine the difference of declination, we require the angular diameter of the ring. This must be found previously by observing the number of seconds that an equatorial star takes to cross the diameter: let it be n, then $15n$ is the value of the diameter in seconds of arc.

Now if T and t be the times required by the stars to describe the chords BC and bc, and if δ be the known declination of one of the stars, the angular lengths of the chords will be approximately (Art. 68).

$$15T\cos\delta \text{ and } 15t\cos\delta.$$

The chords and diameter being known, we can easily obtain the angular distance of each chord from the centre; and their difference or sum, according as they pass on the same or opposite sides of the centre, will be the difference of declination of the two stars.

The Sextant.

111. The sextant does not properly belong to a fixed observatory; but its simplicity, its small size, its accuracy, and, still more especially, its requiring no fixed support, render it of the highest value to the navigator. For a full description of this useful instrument and its adjustments, we shall refer to Chauvenet's *Astronomy*, and to Harbord's *Glossary of Navigation*.

The sextant determines the angular distance between two visible points (as two stars), or the angular altitude of a body above the visible horizon. The principle of its construction depends on the known consequence of the law of reflection that, "when a ray is reflected in one plane at each of two plane mirrors its deviation is double the angle between the mirrors." One mirror is attached to the moveable radius

of a circular sector and another to a fixed radius, both mirrors being perpendicular to the plane of the sector. Then if the moveable arm be turned until the image of a star after two reflections be made to coincide with that of a star seen directly (one-half of the fixed mirror being left unsilvered for the purpose), the angle between the mirrors, and hence the angle between the two stars, can be indicated by the position of the moveable arm among the graduations of the arc.

Double Image Micrometers.

112. As the accurate measurement of small angular magnitudes is of the highest importance, we shall describe two other instruments for the purpose, based on principles entirely different from those of the spider-line micrometer described in Art. 72.

113. *Dollond's Double Image Micrometer.* When a telescope is turned towards a distant object, a circular disc for instance, an image is formed at the focus. Each element

Fig. 1

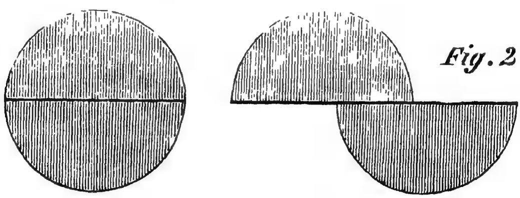
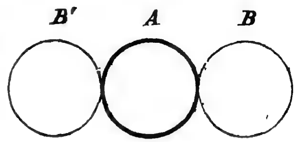


Fig. 2

of the surface of the object-glass gives a complete image of the object, the brightness only increasing with the number of elements employed. If then the object-glass be cut in two, and the two parts kept in their proper places (fig. 1), only one image of the disc will be seen. But if one-half be made to slide along the other, as in fig. 2, two perfect images of the disc will appear, more or less overlapping or separated, according to the distance through which the half-lens has

moved. This sliding motion of the half-lens is given by a screw with a graduated head. Suppose the two images of the disc to be separated till they are just in contact (as A and B), the one image will have moved from the other a distance equal to the diameter of the disc; and if a new contact be established by passing the moveable image to the other side of the fixed one, the whole change, as recorded by the screw-head, will correspond to twice the diameter.



Therefore, supposing the values of the screw-head graduations to have been established, by measuring discs of known sizes, placed at known distances, we shall be able to measure the angular breadths of other bodies, such as the sun and planets.

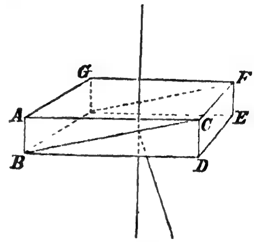
The angular distance between two stars a, b, can be measured in the same way, but the object-glass must firstly be turned till the line of separation of its two halves is in the direction of the line joining the two stars.

Two pairs of stars, a_1b_1, a_2b_2, will become visible by turning the screw, and if coincidences be established between a_2 and b_1, and again between a_1 and b_2, the difference between the two readings will be twice the angular distance between the two stars.

114. *Rochon's Double Image Micrometer.* It is well known that when a pencil of common light passes through certain crystalline substances it undergoes a bifurcation, and two pencils are produced. In uniaxal crystals, such as Iceland spar, one of the pencils is refracted as it would be through ordinary transparent substances, and is called the ordinary pencil; the other, called the extraordinary pencil, undergoes a refraction which varies with its position relatively to a certain line called the axis of the crystal. In one case the

pencils do not separate: this is, when incident perpendicularly on a surface which is itself perpendicular to the axis of the crystal.

Now, if we take two triangular prisms of Iceland spar, *ABCG* and *BCDE*, which, by juxtaposition, form a rectangular parallelopiped *AE*, and if they be cut in such a manner that the axes of the two crystals may be parallel to *AB* and *DE* respectively, then the light, incident on the face *ACFG*, will be parallel to the axis of one crystal and perpendicular to that of the other. No separation, therefore, will take place in the first crystal; but, on entering the second, the two pencils will proceed in different directions inclined to one another at an angle depending on the angle of the prisms.



115. Let *A* (fig. *a*) be the object-glass of a telescope, *PQ* a distant object subtending a small angle, *pq* its image formed at the principal focus. If between the object-glass and the image we introduce the system of crystals above described, with the first surface perpendicular to the axis, the ordinary pencils will proceed without sensible interrup-

Fig. a

Fig. b

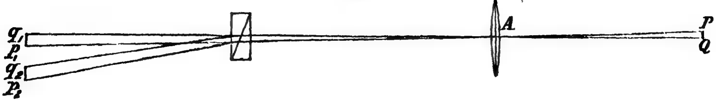
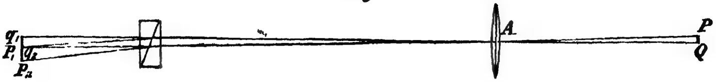


Fig. c



tion, and, provided the opposite faces be parallel, an image p_1q_1 (fig. *b*), will be formed at the focus as before; but, on the pencils entering the second prism, the bifurcation will take place, and a second image p_2q_2, formed by the extraordinary rays, will be obtained.

The distance between the two images will increase or diminish as the prisms are moved from or towards the focus; and therefore by sliding the framework, which carries the crystals backwards or forwards, we can bring the two images to be exactly in contact (fig. *c*).

It is clear then that there will be a connection between the position of the prisms, when the images are just in contact, and the magnitude of the image pq, and therefore also of the angle PAQ which the distant object subtends. If then we observe discs of known sizes, at known distances, we may graduate the telescopic tube in such a manner that the position of the prisms may at once give the angle subtended.*

* M. Arago has suggested a modification of Rochon's micrometer to obviate some slight inconveniences which attend its use. (Arago, *Ast. Populaire*, vol. ii. p. 77.)

CHAPTER VII.

THE SUN.

The Ecliptic.

116. THE observer may now be supposed to possess instruments which will enable him to make observations with any required degree of accuracy, and we shall proceed to examine the movements of the sun, and of those other heavenly bodies which, unlike the stars, have, or seem to have, a motion of their own, independent of the diurnal motion. This diurnal motion, common to all the bodies, is, as we have seen (Chap. II.), only apparent, and belongs to the earth alone.

If we take a celestial globe (Art. 16) on which the stars are all marked in their proper relative places, as also the poles, the equator, and some of the parallels and declination circles, the paths of the sun and of the planets will be conveniently traced by marking their positions on this globe each day.

To the ancients the determination of the exact position of the sun, relatively to the stars, was a problem of considerable difficulty, because the strong light of the sun hinders the stars from being seen at the same time, and the stars become visible only after the sun has disappeared below the horizon.

Hipparchus employed the moon as a link to connect the sun with the stars:—While the sun was above the horizon he determined its position relatively to the moon, and as soon after sunset as the stars became visible, he, in the

same manner, connected the moon with them; then, making allowance for the change of position of the moon during the interval, he was able to deduce the sun's place among the stars. Tycho Brahé substituted the planet Venus for the moon with great advantage, not only on account of its slower and more uniform motion, but especially because the greater distance of the planet considerably lessened the chance of error.

With the transit, the mural, and the clock, we avoid all these difficulties, and obtain results not only with greater facility but also infinitely more accurate.

117. Very rough observations will soon shew that the sun's daily path is constantly changing. This will be seen by noting the points where he rises and sets, the time he is above the horizon, and the greatest height he attains. These vary day by day, and hence the sun's declination varies also; for otherwise, like the stars, he would rise and set always in the same points, and attain the same meridian altitude every day.

That the sun has a progressive motion from west to east among the stars may be inferred from the following facts:—Those stars which are seen near the western horizon shortly after sunset on any evening will, each successive day, remain a less time visible, and after a few evenings will disappear altogether in the strong sunlight, other stars more to the eastward having taken their places. On the other hand, a star in the east, which rises a little before the sun, so as just to be visible, will, each succeeding morning, be seen for a longer time, and attain a greater altitude above the horizon, before the sun's rays overpower it and render it invisible.

We shall see further on that this motion of the sun, like the diurnal motion of the heavens, is only apparent, and is also due to an actual motion of the earth—a motion

of translation—which carries the earth, round the sun, independently of the earth's rotation on its own axis.

Sun's Semi-diameter.

118. When we speak of the path of the sun, we mean the path of the centre of his disc, which is a definite point, whereas the disc itself is a circle subtending an angle of about half a degree. As there is no mark by which the centre can be recognised, we are obliged to observe some point in the perimeter, and allow for the difference between that and the centre. We, therefore, here require the sun's semi-diameter, that is, the angle subtended at the eye of the observer by two lines, one directed to the centre of the sun, the other to a point in the edge. This may be determined by observations with some one of the instruments for measuring small angles, the spider line micrometer or the double image micrometer, as described in Arts. 72 and 112.

The magnitude of the semi-diameter is found to be the same in all directions round the centre,* which shews that the disc is circular; but if the observations be made at different times during the year, the values obtained will be different; a slow decrease taking place from the 31st of December to the 1st of July, and a slow increase during the second half of the year.

The maximum value (on the 31st of December) is 16′17″·8.

The minimum value (on the 1st July) ... 15′45″·5.

The changes are continuous throughout the year, but at corresponding dates in different years the same values recur. These values have been observed with the greatest care, and tabulated so as to be available for future use. (*Naut. Alm.*, p. II. of each month).

* Provided the sun have sufficient altitude to free it from the effects of the large refractions to which it is subject when near the horizon.

To find the Sun's Declination by Observation.

119. At the moment when the sun's centre is crossing the meridian, let the mural telescope be turned to it so as to make either the lower or the upper limb run along the fixed horizontal wire. The corresponding reading of the circle, increased or diminished by the semi-diameter, is the reading of the sun's centre. The difference between this and the *polar-point* (Art. 100) gives the polar distance of the sun, the complement of which is the declination.

Instead of the polar-point, we may use the zenith-point when the latitude of the place is known. The difference between the reading of the sun's centre and the zenith-point gives the zenith-distance, and the difference between the latitude and zenith-distance when the zenith and pole are on the same side of the sun, or their sum when they are on opposite sides, will be the declination.

To find the Difference of Right Ascension between the Sun and any Fixed Star.

120. On reference to Arts. 16 and 17, it will be seen that the difference of right ascension of any two bodies is measured by the interval in time between their transits across the meridian, as given by the sidereal clock.

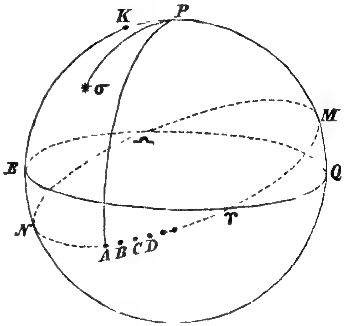
Hence, if, by means of the transit and sidereal clock, both the times be observed when the first and second limbs of the sun cross the meridian, the mean of the two times will give the instant when his centre is on the meridian; and if this be compared with the time when some chosen star is crossing, the interval, reduced to degrees, will be the difference of their right ascensions, that is, the angle between their declination circles.

If the clock gain or lose uniformly, we must multiply the interval so found by $\frac{24}{24+x}$, where $24+x$ is the number of hours marked by the clock between two successive transits of the same star.

Determination of the Sun's Annual Path. Ecliptic. First Point of Aries.

121. Now, on the globe which contains the representation of the starry heavens, let us mark, day by day, the position of the sun at noon by means of his declination, and of the angle his declination circle makes with that of a known star, as determined by the preceding observations.

Thus, supposing σ to be the star selected, P the pole, EQ the equator, make an angle σPA equal to the first difference of right ascension, and mark off PA equal to the corresponding polar distance of the sun, A will represent the first position of the sun. Let B, C D, &c., obtained in the same manner, be the successive positions on the second, third, fourth, &c. days of observation. It will be found that all these points will arrange themselves on a great circle cutting the equator in two opposite points ♈ and ♎, and inclined to it at an angle (ω) of about 23° 27′ 30″.



122. This great circle, the plane of which contains the sun's yearly path, is called the *ecliptic*, and the angle (ω) it makes with the equator is the *obliquity of the ecliptic.*

Its intersections with the equator are called the *equinoctial points*, one (♈) the *first point of aries*, the other (♎) the *first point of libra.*

The sun will be found in the first of these points about the 21st of March, and in the other about the 23rd of September, his declination being then 0° and his polar distance 90°.

The two points M, N of the ecliptic, equidistant from the

equinoctial points, are called *solstitial points;* and if K be the pole of the ecliptic, the great circle through K and P will pass through the solstitial points, and be perpendicular both to the equator and to the ecliptic. It is called the *solstitial colure.*

When the sun reaches the solstitial points, he has his greatest declination 23° 27½′ north or south. This occurs about the 22nd of June and the 22nd of December. Therefore, from the 21st of March, when the declination is 0°, to the 22nd of June, the sun has an increasing north declination; from the 22nd of June to the 23rd of September, the north declination decreases to zero; from the 23rd of September to the 22nd of December the declination is south and increasing, and it then decreases until the 21st of March.

Variations in the Length of the Day.

123. The combination of this annual motion of the sun, with the diurnal motion of the earth, will explain all the phenomena of the seasons, and of variations in the length of the solar day a. particular places. This we shall proceed to shew.

Let fig. *a* be the globe on which are delineated the stars, the pole, the equator, and the ecliptic; and let fig. *b* represent the celestial sphere of an observer.

We shall find it convenient thus to use two distinct figures; on fig. *a* we shall trace the annual motion, and on fig. *b* the apparent diurnal paths due to the earth's rotation; fig. *a* will be common to all observers, but fig. *b* will vary with the observer's latitude.

If the sun were fixed among the stars, he would, by the effect of the earth's rotation, describe a parallel—higher or lower above the horizon, and visible for a greater or less time, according to his declination, and to the position of the observer—but in his progress along the ecliptic he changes his declination continually; and not only is the

apparent path of each day different from that of the day before, but it ceases to be circular; and all the daily paths together, if we include those portions which are hidden below the horizon, form one continuous spiral curve.

The sun describes 360° in $365\frac{1}{4}$ days—his advance along the ecliptic in therefore about 1° per day; and the daily change is his declination is still slower, since it only varies in the same time from $23\frac{1}{2}$° south to $23\frac{1}{2}$° north, and back again. In the following explanations it will be sufficient to assume that, throughout each day, the sun's declination is constant, retaining the value which it has at sun-rise; that is, we shall consider his daily paths to consist of a series of parallels instead of a spiral.

Fig. a

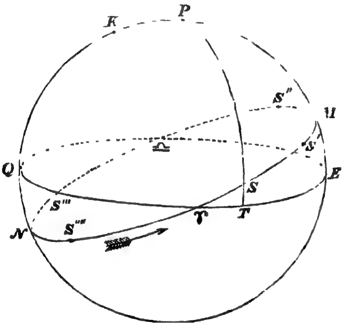
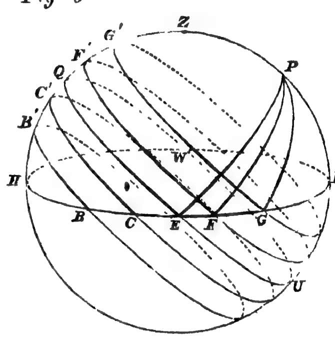


Fig b



124. The first case we shall consider will be that of an observer in the northern hemisphere, whose latitude is somewhere between 23° $27\frac{1}{2}'$ and its complement 66° $32\frac{1}{2}'$. Fig. *b* is adapted to this supposition: *Z* is the zenith, *P* the pole, *EQ* the equator, and *HR* the horizon. The latitude is *ZQ* or, its equal, *PR*.

Let us begin on the 21st of March, when the sun is at ♈ (fig. *a*) and his declination is 0°. He will rise at *E*, the east point of the horizon (fig. *b*), his bour angle being then *EPZ*, or 90°; during the day he will describe the equator *EQW*, and will set at *W*, the west point, where

his hour angle will again be 90°. The day and night will then be of equal duration, whence the term equinox applied to this epoch of the year. As marking the commencement of spring, it is further distinguished as the vernal equinox. On that day the sun attains an altitude HQ equal to the co-latitude.

A few days later, the sun will have advanced to S (fig. a), having a north declination ST or δ; and if the parallel FF' be drawn (fig. b) at a distance δ from the equator on the north side, this will be the diurnal path of the sun on that day. At sun-rise the sun will be at F to the north of east, his hour angle will be FPZ, which $> EPZ$, that is, $> 90°$; therefore the day will be longer than the night. His altitude at noon will also have increased, being HF', which $= co\text{-}lat. + decl.$

The length of the day and the meridian altitude will thus go on increasing until the 22nd of June, when the sun reaches the solstitial point M (fig. a). Here his declination $= ME = \angle M\Upsilon E = \omega$. If we make QG' equal to ω (fig. b), the parallel GG' will represent the sun's path on that day, the meridian alt. $= co\text{-}lat. + \omega$, the hour angle at sun-rise is GPG', and the azimuth HG.

These are their greatest values; for, after this, as the sun continues his course along the ecliptic, fig. a shews that his declination begins to decrease, and therefore his daily path (fig. b) will re-approach the equator. For several days before and after the maximum, the change of declination will be very slight, and during this time there will be no perceptible change in the diurnal path—to the eye, he will rise at the same point of the horizon, reach the same meridian altitude, and set at the same point. Hence the term *solstice* applied to this period of the year, and *solstitial point* to the point M of the ecliptic. This is the *summer solstice.*

During the next three months the sun will pursue his

course from M to ♎ (fig. a), his distance from the equator gradually diminishing. His diurnal paths will therefore be a repetition, but in an inverse order, of his paths during the preceding three months.

We thus reach the autumnal equinox, the sun being again in the equator, and the night and day of the same length. After this the sun will pass to the south side of the equator; his daily paths, such as CC', BB', (fig. b) will be on the further side of the equator from the elevated pole, his points of rising and setting will more and more recede from E and W towards H, the hour angle at sun-rise will be less than a right angle and decreasing, and the altitude at noon each day less than the preceding, until the sun reaches his furthest distance from the equator on the south side at the solstitial point N (fig. a). If BB' be the corresponding diurnal circle, the meridian altitude

$$HB' = HQ - QB' = co\text{-}lat. - \omega.$$

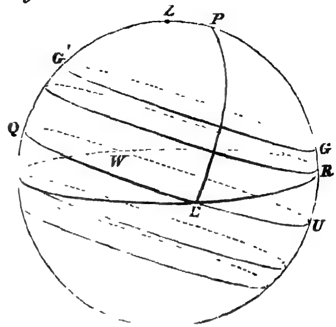
From the winter solstice, which occurs about the 22nd of December, the sun returns to his starting point ♈, the south declination gradually decreasing, and the sequence of diurnal circles from the 22nd of December to the 21st of March being in an inverse order, the same as from the 23rd of September to the 22nd of December.

125. Suppose we travel towards the north pole until our latitude exceeds $66° 32\frac{1}{2}'$. Let fig. c be the celestial sphere adapted to our new position, fig. a remaining the same as before.

The pole will be here at a distance ZP from the zenith less than $23° 27\frac{1}{2}'$ or ω. For the latitude ϕ is the complement of ZP. Therefore ZP (which $= HQ$ or RU) $< \omega$.

At the vernal equinox, the sun being in the equator will describe EQ, the hour angle EPZ at sunrise being, as before, a right angle; and, consequently, the night and the day of equal length.

Fig. c



The increase in the length of the day will be more rapid than at the previous station, and when the sun has reached a point S' (fig. a) in ΥM, where his declination is $90° - \phi$ or RU (fig. c), he will not set at all, but at midnight will just graze the horizon at R, and then re-ascend, remaining now continually above the horizon during the whole time he takes to move from S' to the corresponding point S'' on the other side of the solstice M.

At the solstice itself the path, during twenty-four hours, is the small circle GG', the greatest altitude being HG', or $\omega + co\text{-}lat$, and the least RG', or $\omega - co\text{-}lat$.

After passing S'' the sun will again begin to set, and at ♎ the day and night will be of equal length. The days will continue to decrease until the sun's south declination at S''' is $90° - \phi$. Then his diurnal path will only just graze the horizon at H, and a period of continuous night will set in from S''' to S'''', corresponding to that of continuous day about the summer solstice.

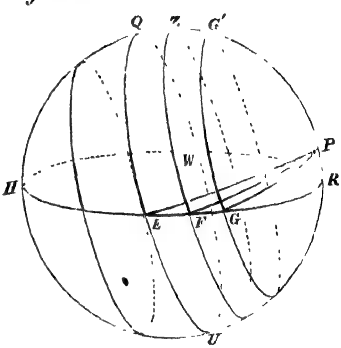
The subsequent re-appearance of the sun above the horizon, and the gradual increase of daylight, will be easily understood without further explanation.

126. The points S' and S'', which have a declination $90° - \phi$, will recede from M as ϕ increases, and therefore the period of perpetual day will increase with the latitude, attaining its maximum when the observer is at the pole; for there the equator and the horizon coincide, the sun remains visible while on the north side of the equator from the vernal to the autumnal equinox, and this day of six months is followed by six months of perpetual night.

127. If we travel towards the equator until our latitude ϕ is less than 23° $27\frac{1}{2}'$, the phenomena will again be different. Fig. *d*, where ZQ or PR is this lower latitude, will be adapted to this case; and we must, as before, take fig. *a* in connection with it.

At the equinox, the sun will describe the equator EQ, and the day and night be twelve hours each as before. Also as the sun's declination increases, his meridian altitude will increase until he has reached that point S of his orbit (fig. *a*) where his declination is equal to ϕ the latitude. On that day his diurnal path will pass through Z the zenith, and the sun at noon will be vertical.

Fig. d



On the next and subsequent days, his declination still increasing, his path will cross the meridian between the zenith and the pole; the hour angle at sun-rise, and therefore the length of the day, will still increase, but his meridian altitude will decrease.

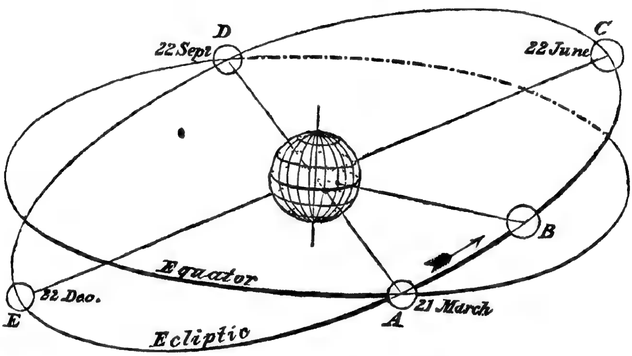
The solstice M being passed, the days will begin to decrease, and once again the sun will culminate in the zenith; then, until he has his greatest south declination, his meridian altitude will decrease, and the days get shorter. The return to ♈ will be attended with corresponding variations, which need not be traced.

128. At the equator itself, the pole will be in the horizon, and the hour-angle at sun-rise a right angle; therefore the days will always be twelve hours long; and as the celestial equator passes through the zenith, the meridian altitude will always be the complement of the declination.

129. It will be obvious, by an examination of figs. *b*, *c*, *d*, that the variations in the duration of daylight throughout the year will be greater the higher the latitude; and it can also be easily inferred from these figures, that the longest summer day and the longest winter night, at any place, are of equal lengths (see Chap. XIV.).

In the southern hemisphere, the phenomena of night and day will be the exact counterpart of those in the northern.

130. The foregoing articles explain the gradual change in the diurnal phenomena observed at particular places on the earth's surface—the general explanation for the whole earth will be best understood by an examination of the accompanying figure, where *A*, *B*, *C*, *D*, *E* represent successive positions of the sun in his yearly path.



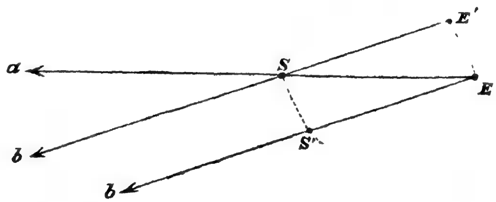
The arrow represents the direction of the sun's motion, and the earth at the centre turns in the same direction about its axis. Once a day, each terrestrial meridian comes in its turn opposite to the sun, and it is noon at all places in that meridian. To one-half of the earth there is daylight, to the other, night. If the centres of the sun and earth be joined by a straight line, the point where this line meets the earth will at that instant have the sun in its zenith, and will obviously be the centre of the illuminated part of the earth; and the line of demarcation between light

and darkness will be approximately* the great circle, of which this central point is the pole. It will be easily seen, that at the time of the equinoxes, when the sun is at *A* or *D*, the boundary passes through the two poles; and that at the solstices when the sun is at *C* or *E*, one pole with $23\frac{1}{2}°$ around it in all directions will have continuous daylight, and the other continuous night.

Annual Motion of the Sun replaced by a Motion of the Earth.

131. We shall, in the next place, proceed to shew that this daily displacement of the sun—this shifting of his position among the fixed stars, which constitutes what we have called the annual motion—may be explained by a corresponding motion of the earth round the sun at rest, provided we assume the distance of the stars from the earth to be infinitely great in comparison with that of the sun.

Let *E* and *S* represent the earth and sun. An observer



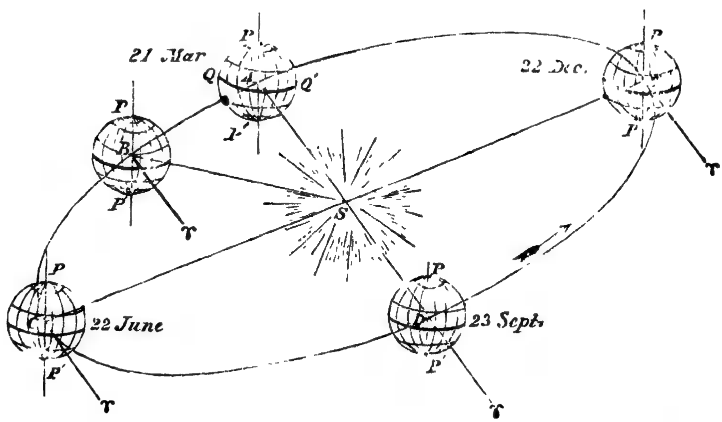
at *E* sees the sun *S* in the direction of some star *a*. If the sun move to *S'*, the observer will then see him in the direction of some other star *b*. But the very same star *b* would also be the sun's direction, if we suppose the sun to have remained stationary at *S*, and the earth to have moved to *E'*, found by making *SE'*, in the line *bS* pro-

* We say approximately, because, on account of the sun being much larger than the earth, the cone which envelopes the two bodies will touch the earth in a small circle beyond this great circle. The effect of refraction will still further diminish the dark portion.

duced, equal to $S'E$. Moreover, if the stars are so distant that the two lines ES', $E'S$, which lead to the same star, may be considered parallel as well as equal, the path EE' round S will be exactly equal and similar to the path SS'. Hence, whichsoever supposition be the true one, it is obvious that the apparent motion may nevertheless be explained as a consequence of the other. If we adopt the hypothesis of the earth's motion—and we shall see presently the arguments in its favour—we must suppose that during its annual motion about the sun its axis remains parallel to itself, in order that the polar distances of the stars may remain constant.

The figure below will exemplify this motion of the earth, and will account for the observed changes of right ascension and declination of the sun.

Let ABC... be the path which the earth would have



to describe about S the sun, in a plane inclined at $23° \; 27\frac{1}{2}'$ to the earth's equator, PP' the earth's axis, P the north pole.

Let A be the position of the earth on the 21st of March when the plane of the equator QQ' passes through the sun.

The line joining the centres of the two bodies at any time will, by its intersection with the surface of the earth,

determine that place which has the sun in its zenith at that moment. This will be some place in the equator when the earth is at *A*, and it would happen in turn to every place on the equator as the earth revolved about its axis, if we neglected the motion of translation of the earth during the twenty-four hours. Neglecting this motion would be the exact equivalent of the supposition we made, for simplicity of explanation, in Art. 123, where we assumed the sun's declination to remain constant during twenty-four hours.

Some few days or weeks afterwards suppose the earth to have arrived at *B*. The plane of its equator, which is always perpendicular to *PP'*, will now be below the sun; the line joining the centres will make an acute angle with the axis *BP*, and the sun will be vertical to a place north of the equator, whose latitude is equal to the declination of the sun, each being the complement of the angle *PBS*.

If *SC* be drawn at right angles to *SA*, when the earth reaches *C*, the line *CS* will make a greater angle with the plane of the terrestrial equator than in any preceding or succeeding position, and the sun's north declination will have attained its maximum 23° 27½'. This will be at the summer solstice, about the 22nd of June.

From *C* to *D* the declination will decrease, and at *D* be zero, the plane of the equator again passing through *S*. This will be the autumnal equinox. *E* will be the position of the earth at the winter solstice when the sun attains his greatest south declination.

132. The series of changes in the sun's declination is thus seen to be a consequence of the earth's motion, and it will also be easily seen that the same motion produces the gradual increase of right ascension.

At *A* the earth sees the sun in the direction *AS*♈, and, in all subsequent positions, the plane of the earth's equator remaining parallel to itself, the lines of intersection with

the plane of the ecliptic, viz. $B\Upsilon$, $C\Upsilon$, &c., will all have parallel directions, and the sun will seem to have moved through the angles ΥBS, ΥCS, &c., which exactly correspond to the angles which the arcs AB, AC, &c., in the supposed motion, fig., p. 102, subtend at the centre of the earth.

Arguments in favour of the Earth's Motion.

133. The following are the chief arguments in favour of the supposition that it is to the earth, and not to the sun, that the motion of translation belongs.

(1) It furnishes a simple explanation of the stationary points and of the retrograde motion of the planets (see Chap. XXII.). These may also be explained on the other hypothesis; but the ancient astronomers, who assumed the earth to be stationary at the centre, were driven to a complex system of epicycles to account for the phenomena.

(2) It is in accordance with observation that the distance of the sun and the length of the year have just that relation to one another which they should have if we suppose the earth to be a planet revolving round the sun and subject to the same laws as the other planets.

(3) The strongest proof is furnished by the "aberration of light." This admits of easy explanation if we suppose the earth to have a motion of translation, but no explanation has been given on any other hypothesis (see Chap. XVII).

134. Although the annual motion really belongs to the earth, and not to the sun,* we shall sometimes find it con-

* When we say *not to the sun*, we do not mean that the sun is absolutely fixed, for we shall have occasion to conclude that the whole solar system—the sun with its attendant planets—has a motion of translation in space. But the motion we are considering in the text is the internal relative motion of the parts of the system.

venient to retain the latter supposition when it simplifies, without vitiating, the validity of the reasoning. This we shall do in Chap. XII., when explaining the "equation of time," where we shall attribute the annual motion to the sun and the diurnal motion to the earth.

CHAPTER VIII.

THE SEASONS.

135. The two equinoctial and the two solstitial points divide the ecliptic into four equal parts, but, owing to inequalities in the sun's velocity, they are not described in equal times. See Chap. XI.

These unequal periods are called *Seasons*, and are distinguished as *Spring*, *Summer*, *Autumn*, and *Winter*.

Spring commences when the sun is at ♈ (fig., p. 95), and lasts till he reaches the solstitial point M; the summer, while he goes from M to ♎, autumn from ♎ to N, and winter from N to ♈. Their lengths are at present—

Spring	92	days	21	hours
Summer	93	"	14	"
Autumn	89	"	$17\frac{3}{4}$	"
Winter	89	"	1	"

136. The discussion in Art. 124 shews that for an observer in the northern hemisphere, between the parallels of $23° 27\frac{1}{2}'$ and $66° 32\frac{1}{2}'$, the days in spring and summer are longer than the nights, and the reverse in autumn and winter; but spring differs from summer, and autumn from winter, in the mode in which the duration of daylight changes—increasing in the first and last, decreasing in the other two.

The four seasons are still further characterised by their temperatures. The amount of heat received from the sun depends on the time he remains above the horizon, and,

in a still greater degree, on the altitude he attains during the day. Hence spring and summer must be warmer than autumn and winter.

But the heat of summer must be greater than that of spring, although the same altitudes and the same lengths of days occur; because, for some time after the solstice the quantity of heat received each day, though decreasing, will be in excess of the quantity lost or used up, and the temperature will go on increasing; just as the greatest heat of any day is not until about two hours after noon, for only then is an equilibrium obtained between the gain and the loss.

For a similar reason the winter quarter will be colder than autumn, just as the coldest hour of the night is about two or three hours after midnight.*

All this is in exact agreement with what is observed in England, and in all places between the parallels of 23° 27½′ and 66° 32½′ of north latitude.

137. If we go to higher latitudes (Art. 125), the rays of the sun impinge more obliquely upon the earth, their heating power is weakened and the mean temperature of the year is lowered. During the summer months, the continual presence of the sun for many days above the horizon compensates to some extent for this loss of heating power, so that in some places there is a warm, though brief, summer. On the other hand, the long winter nights reduce the temperature so as to produce that intense cold which covers the seas with ice, and renders the land unfit for permanent habitation beyond the latitude of 70°.

138. Between the equator and the parallel of 23° 27½′ we have a belt or zone where the conditions are very different.

On the equator itself the days are always 12 hours long, but the sun at noon is never more than 23° 27½′ from the

* The differences (hereafter to be noticed) in the sun's distance from the earth have but slight influence on the amount of heat we receive from him.

zenith, and twice a year, that is at the equinoxes, is exactly vertical at noon. We have here the main conditions for a high temperature during the whole year, and the seasons, instead of being marked by contrasts of temperature, as in the regions distant from the equator, are marked by contrasts of humidity, and consist of two rainy seasons and two dry seasons.

To places off the equator, but within 23° 27½′ from it, the same remarks will apply—twice a year, but at unequal intervals, the sun will be vertical, and his distance from the zenith at noon will, even at midwinter, be less than 47°. The climate is characterised by extremely elevated temperature at all times.

139. It is obvious that what has been here said of the northern hemisphere will be equally applicable to the southern hemisphere, except that the phenomena in southern latitudes, though precisely the same, occur at exactly opposite epochs of the year. When it is summer in the north it is winter in the south, the beginning of autumn in one hemisphere is the beginning of spring in the other.

Zones of the Earth.

140. The part of the earth included between the parallels of 23° 27½′ south and north of the equator is called the *Torrid Zone*, and the bounding parallels are called *Tropics:* the *Tropic of Cancer* on the north side and the *Tropic of Capricorn* on the south.

The parallel of 66° 32½′ north latitude is called the *Arctic Circle*, and the corresponding one in south latitude the *Antarctic Circle.*

The regions round the two poles, bounded by the arctic and antarctic circles respectively, are called the *Frigid Zones.*

The two belts of the earth comprised between the limits of the torrid and the frigid zones are called the *Temperate Zones.*

The reasons for these names will be understood from the previous explanations, but it should be remembered that as we approach the limits of the temperate zone, the character of the climate differs less and less from that peculiar to the neighbouring torrid or frigid zone.

Additional Remarks on the Temperature.

141. The temperature has been spoken of as varying with the duration of the sun's presence above the horizon and with his meridian altitude; but if these were the only causes of disturbance, we ought to find exactly the same temperatures at all places in the same latitude—which we know to be far from being the case.

The sea is found to preserve a much more uniform temperature than the land, and therefore islands and places on the coasts will have their extremes of heat and cold much nearer to each other than in the interior of the continents. The cold of winter and the heat of summer depend more on the greater or less distance from the ocean than on the latitude. Thus, London and Irkutsk (Siberia) are nearly in the same latitude, but while London has a mean range of only 24°·3 Fahrenheit, Irkutsk has 61°.

	Mean Temperature.			Mean of Year.
	Winter.	Summer.	Diff.	
London Lat. 51½° N.	37·8 F.	62°·1 F.	24°·3 F.	49°·6 F.
Irkutsk 52¼° N.	− 0·2	60° 8	61°·	31°·6

This great difference is almost wholly thrown on the winter temperature, for the summer heat of London is only about 1° higher than at Irkutsk, whereas the winter cold of the latter place is 38° more severe than that of the former, and the mean temperature of the year is 18° lower.

Probably the principal agent in the mitigation of the cold of our winter is to be found in the prevalent S.W. or Anti-Trade Winds which come to us charged with the

warmth and moisture they have taken up in the inter-tropical regions of the Atlantic. The condensation of the vapour in the form of rain or snow restores the heat which had been expended in raising the vapour in those distant seas.

Ocean currents will also have a very powerful influence in modifying the temperature. To whatever cause it may be due, it appears certain that there is a constant interchange of water between the Tropics and the Arctic regions. The large number of observations made by Dr. Carpenter during the last few years have led him to adopt the doctrine of a general oceanic vertical circulation sustained by opposition of temperature alone.* He infers that there is a continual bottom outflow of Polar water, due to increase of density and pressure, along the deepest channels of communication with other Oceanic basins. And as a necessary consequence of the reduction of level which must result from this outflow in the Polar area, there will be a continual surface indraught for its restoration; and thus a movement will take place in the surface-water from the Tropics towards each Pole.

The rotation of the earth from west to east must be considered in connection with this movement of the waters. The nearer the equator, the greater will be the eastward velocity of a body; the warm surface current will, therefore, as it moves towards the poles, travel eastward more rapidly than the parts of the earth which it successively reaches, thereby acquiring an eastward direction; while for a like reason, the cold polar current will flow towards the equator and westward.

A north-easterly drift of this character is found carrying the waters of the mid-Atlantic towards the British Islands, and its warming influence is felt even as far north as Spitz-bergen and Nova-Zembla. A cold arctic current on the

* See an article by Dr. Carpenter in *Good Words*, Jan. 1873.

other hand, with a westerly tendency, sweeps along the eastern coasts of Greenland and of North America as far south as 40°; so that while the harbour of Hammerfest (Norway), in 70½° north latitude, is free from ice all the year round, the opposite coast of America is blocked up as far south as 50° north latitude during the whole winter, and a great part of the early summer.

The same remarks will apply to the west coast of America and the opposite shores of Asia, but in a much less marked degree, for the shape of the continent gives no outlet for the cold current from the Arctic Ocean through Behring's Straits, although the north-easterly warm current from Japan is felt along the west coast of British Columbia, and even beyond the Straits, further up on the north coast of America. The opposite east coast of Asia has, however, a higher temperature than the corresponding latitudes of the east coast of America.

The northerly drift above spoken of does not begin near the equator. Between the parallels of 40° north and south of the equator, surface currents are found describing circuits of immense extent, with central expanses of quiescent water covered with marine vegetation, and undisturbed except by winds and tides. These equatorial currents are probably due to the propelling action of the trade and anti-trade winds; but, whether due to these alone or to these combined with other forces, the result is a continuous motion of an enormous body of water. If we follow the equatorial current of the North-Atlantic, we find it travelling westward from the coast of Africa towards the north coast of South America, where, being driven by the conformation of the land into the Gulf of Mexico, it is turned northward, and the *Gulf-stream*, as it is now called, flows through the Florida channel and outside of Cuba, a warm current with a speed of from two to four miles an hour. Taking now a north-easterly course it gradually leaves the American shore, and,

propelled perhaps by the anti-trade winds, a portion of it joins the N.E. drift spoken of before, and adds its warmth to the other causes which render our winter climate milder than that of corresponding places in the same latitude. There is now reason to believe that the larger portion of the Gulf-stream, after reaching the meridian of the Azores, turns southward and completes the circuit by rejoining the current off the coast of Africa.

In south latitudes, the immense expanse of water keeps the temperature much more nearly dependent on the latitudes alone.

Trade Winds.

142. What has been said about the ocean currents will prepare for the explanation of the *Trade Winds.*

The air in the torrid zone, being heated by the land and sea, expands and rises, forming an upward current which must be fed by the lower air of the higher latitudes, and a current therefore sets in towards the equator from the temperate zones. The warm air which has risen spreads over the other, and thus a constant interchange takes place, causing an upper current from the equator towards the poles; but this upper current being gradually cooled will sink again to the earth somewhere in the temperate latitudes.

There will, therefore, be in the torrid zone an under current of air setting towards the equator; but this air, having less easterly velocity of rotation than the portions of the earth which it successively reaches, will lag behind, and the current which starts from the northern hemisphere as a *northerly* wind, will, as it advances southwards, gradually pass into a *north-easterly* wind, and in some cases even become an *east* wind.

This constant current, which blows nearly all the year round, is called the *N.E. trade-wind;* and a corresponding current called the *S.E. trade-wind*, advancing from the

southern hemisphere towards the equator, veers gradually round, becoming more and more easterly.

The upper current of air, which sets north and south from the equator, having the same easterly velocity as the equator, becomes, where it again reaches the earth, the *anti-trade* wind, the prevailing *south-westerly* wind of our latitudes, and a *north-westerly* wind in the southern hemisphere; but the position of the sun, mountain ranges, and accidents of country, will modify these general results.*

143. The trades do not, however, meet and combine in a common easterly direction near the equator. They are separated by a belt some 600 miles in breadth, characterised by heavy rains and calms. In the Atlantic, the N.E. trade-wind is felt from about 25° to 8° of north latitude, and the S.E. trade-wind from about 25° to 2° of south latitude, but these limits vary with the season.

At its southern limit the N.E. trade loses its easterly tendency and the wind gets gradually more northerly; and so at the northern limit of the S.E. trade, the wind shifts into a more southerly instead of easterly direction. Basil Hall, who notices this fact (*Fragments of Voyages*, 2nd series, vol. I., p. 289), gives the following explanation: The friction with the surface of the water destroys the velocity of the trade-wind, so that its westward motion is diminished. The motion towards the equator would also diminish from the same cause, but the original pressure which produces this motion is still acting, and the particles progress towards the equator, and as the successive parallels of latitude do not now increase so rapidly as at first, the westerly motion will be less.

This equatorial belt, where the winds from the north and the south meet and counteract one another, is not stationary,

* The explanation of the Trade Winds, given above, was first proposed by Halley (*Phil. Trans.* 1686, p. 167).

the interior limits of the trades shifting north or south as the sun's declination alters. In the Indian Ocean, the S.E. trade crosses the equator in the summer, and extends as a *S.W. monsoon* to the foot of the Himalayas. In the same way the southern limit of the N.E. trade, during the winter months of the northern hemisphere, not only reaches the equator, but crosses it and appears in the southern hemisphere as the N.W. monsoon (Dové's "*Law of Storms*").

In the 6th vol. of the Smithsonian *Contributions to Knowledge*, Prof. Coffin has collected and discussed an immense number of observations on the winds of the northern hemisphere. From these it may be inferred that the proximity of large continents has considerable influence on the winds. Thus, in the North-Atlantic and in the North-Pacific the trade wind veers gradually round to the east, except close to the west coasts of Africa and of America, where the land draws as it were the wind to it, and gives it a more northerly direction. Along the coast of California, following the trend of the land, it ever becomes north-westerly.

CHAPTER IX.

FINDING THE LATITUDE BY OBSERVATION.

144. THE determination of the latitude is a problem of primary importance in an observatory, and the greatest accuracy, both of calculation and of observation, must be brought to bear upon it.

At sea, it becomes a problem of daily necessity, but the same precision is neither requisite nor possible. The ship's place is constantly changing, and the delicate instruments and methods of observation in use in an observatory must be replaced by others adapted to the unstable position of the observer.

A second of latitude corresponds to a distance of about thirty-four yards, and the latitude of a fixed observatory is determined to within a few *tenths* of a second; but this is after a series of observations extending probably over a number of years. At sea, the latitude can seldom be relied on to within a quarter of a mile, and the nearest half-mile is generally considered sufficient.

145. The methods may be classed under three heads: *meridian* observations, *ex-meridian* observations, and *circum-meridian* observations. Of the following methods the first is the simplest, and in a fixed observatory the most accurate, but it is not adapted to general use at sea, on account of the twelve hours' interval between the observations, which will frequently require one of them to be made in the day-time, when only the powerful fixed telescope of an observatory can perceive the star.

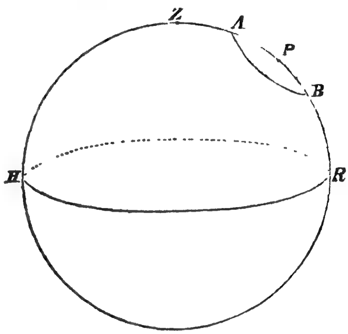
The second method is that in common daily use on board ship.

The third, fourth, sixth, and seventh methods are sea methods, especially useful when clouds or other circumstances interfere with the meridian altitudes.

The fifth method can only be practised in an observatory fitted with a transit in the prime vertical.

Meridian Observations.

146. *First Method.* To find the latitude by observations of the two meridian altitudes of a circumpolar star.



Let Z be the zenith, P the pole,

HR the horizon,

α the altitude AR of a star at upper transit,

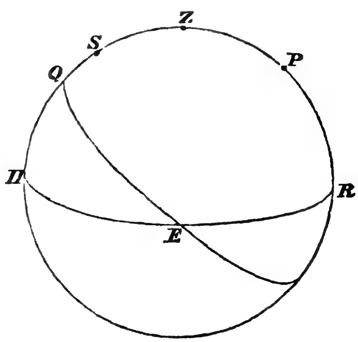
β the altitude BR of the same star at lower transit;

then, $PR = \frac{1}{2}(AR + BR)$,

or, $lat. = \frac{1}{2}(\alpha + \beta)$.

In this method no previous knowledge of the star's declination is required.

147. *Second Method.* To find the latitude by observation of the meridian altitude of the sun, a star, or other body, whose declination is known.



Let S be the sun or star in the meridian, QE the equator,

δ the known declination SQ,

α the observed meridian altitude SH,

$z = 90° - \alpha$, the zenith distance ZS;

then $ZQ = ZS + SQ$, or, $lat. = z + \delta$.

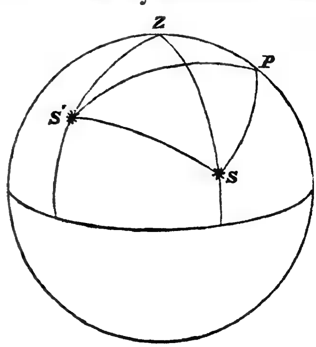
If the object crossed the meridian between Q and H, the formula would be $lat. = z - \delta$; and if between Z and P, $lat. = \delta - z$. These are all included in the single formula $lat. = z + \delta$, provided z be reckoned + when the zenith is north of the object, and − when south; and similarly, north declination +, and south declination −. The latitude will be north or south, according as the result is + or −.

If the altitude is observed when the object is crossing the meridian below the pole, that is, between P and R, it will be easily seen that $lat. = 180^\circ - z - \delta$, and that it must necessarily be of the same name as the declination.

In the practice of this method at sea, it is impossible to ensure that the observation shall be made exactly in the meridian; but the change of altitude being then very slow, no practical difficulty arises. The altitude of the sun or other body is taken a quarter or half-an-hour before the meridian passage, and the observation repeated at short intervals until the maximum is passed, which maximum is then taken for the meridian altitude. The observations are of course made with a sextant, the image of the star or of the sun's limb being brought into coincidence with the visible horizon and the instrumental reading corrected for dip, refraction, &c.

Ex-meridian Observations.

148. *Third Method.* To find the latitude by simultaneous observations of the altitudes of two known stars.



Let S, S' be the two stars,

z, z' their zenith distances,

Δ, Δ' their known polar distances,

α the known difference SPS' of their right ascensions.

In the triangle SPS', the two

sides PS, PS' and the contained angle are known, whence the third side SS' and the angle PSS' may be calculated:

$$\cos SS' = \cos\Delta \cos\Delta' + \sin\Delta \sin\Delta' \cos\alpha \;\ldots\ldots\; \text{(i)},$$

$$\sin PSS' = \frac{\sin\Delta' \sin\alpha}{\sin SS'} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; \text{(ii)}.$$

The three sides of the triangle ZSS' are now known, and the angle ZSS' may be computed

$$\sin\tfrac{1}{2}(ZSS') = \sqrt{\left\{\frac{\sin\frac{1}{2}(z' + z - SS') \sin\frac{1}{2}(z' + SS' - z)}{\sin SS' \sin z}\right\}} \ldots \text{(iii)},$$

then $PSZ \,(= PSS' - ZSS')$ will be known.

Finally, in the triangle ZPS we have two sides and the contained angle, and the third side $ZP =$ co-latitude may be computed

$$\sin lat. = \cos ZP = \cos\Delta \cos z + \sin\Delta \sin z \cos PSZ \ldots \text{(iv)}.$$

This method is not so extensively used by seamen as it deserves to be. It is susceptible of great accuracy, because, during the morning and evening twilight, the horizon is generally well defined, and the altitudes of bright stars can be readily observed. Moreover, so long as the same two stars are used, the values of SS' and PSS' are constant, and may be tabulated for certain pairs of principal stars, leaving only two quantities, ZSS' and ZP, to be determined from the formulæ (iii) and (iv).*

The observations must be simultaneous; but when there is only one observer, proceed as follows:—Take the altitude of S, then the altitude of S', and again the altitude of S, noting the corresponding intervals of time. During the few minutes that this will occupy, the changes of altitude may be supposed to be uniform, and therefore the altitude of S, corresponding to the instant of the observed altitude of S' may be easily found.

* This is done in the "Star Tables" published by Capt. Shadwell, R.N.

149. *Fourth Method.* To determine the latitude by two observations of the altitude of the sun and the elapsed time.

This is in reality the same problem as the last, the declination of the sun at the two times of observation replacing the declinations of the stars, and the elapsed time reduced to degrees, at the rate of 360° for 24 hours, giving the angle SPS', which was the difference of right ascension of the two stars.

A modification, however, arises in the case of the sun, from the fact that his declination changes very little in the interval between the observations, and may be considered the same at each as it is at the middle time between the two.

The triangle SPS' thus becomes an isosceles triangle, and, bisecting it by a perpendicular from P (not drawn in the figure), the two formulæ (i) and (ii) are replaced by

$$\sin \tfrac{1}{2} SS' = \sin \Delta \; \sin \tfrac{1}{2}\alpha,$$

$$\cot PSS' = \cos \Delta \; \tan \tfrac{1}{2}\alpha;$$

the remainder of the solution will proceed as before.

The same solution will obviously apply to two altitudes of the same star and the elapsed time, provided this elapsed time be reckoned in sidereal hours.

150. At sea, when two observations are taken, as in the fourth method, separated by an interval of time, allowance must be made for the change of the ship's position during the interval.

This is practically a very simple operation, because the distance run always subtends a small angle at the centre of the earth.

Let Z and Z' be the zeniths of the two places, and S the sun or star, whose zenith distance ZS was observed at the first place of observation. We have to

determine $Z'S$, the zenith distance which would have been observed at the same instant at the second place.

Let the ship's course by compass, and distance run during the interval, be measured by the usual methods practised at sea. Also let the compass-bearing of S, at the first observation, be noted. The angle θ, between the ship's course and the bearing of S, will be the angle SZZ', and the distance run by the ship in nautical miles will be the number of minutes in the arc ZZ'.

Draw $Z'X$ perpendicular to SZ. Then ZX will be very approximately the difference between SZ and SZ', and, the sides of the triangle $ZZ'X$ being small,

$$ZX = ZZ' \cos\theta;$$

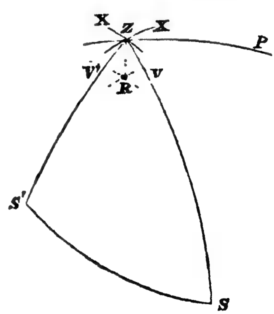
therefore $$SZ' = SZ - ZZ' \cos\theta,$$

and the first observation is reduced to what it would have been if made at the second place.

151. In order to determine the most favourable circumstances for the success of the third and fourth methods, let us examine the graphical solution of the problem.

We shall assume that the declinations and right ascensions of the two bodies, or the declinations and elapsed time in the case of two positions of the same body, are accurately known; and we wish to ascertain under what conditions of observation, errors in the altitudes will have the least weight in the determination of the latitude.

Let P be the pole; S, S' the two bodies, or the two positions of the same body.



With S as pole and a spherical radius SZ, or z, the zenith distance of S, describe an arc ZX. Similarly, with S' as pole, and a spherical radius $= z'$, describe the arc $X'Z$, cutting the former in Z; then Z will be the zenith and PZ the co-latitude.

When the two observations are made with different instruments, the errors of observation not only being unknown, but having no necessary connection, it will be best to select the time when the angle between the two arcs ZX, ZX' approaches nearest to a right angle; for, if the possible error in each observation be θ, the greatest possible error in the value of the latitude will, when the angle is a right angle, be the diagonal of a square, whose side is θ; whereas, if the two arcs cut each other obliquely, the greatest possible error will be the longest diagonal of the parallelogram, whose angle is the same as that between the arcs, and the perpendicular distance between the opposite sides of which equals θ.

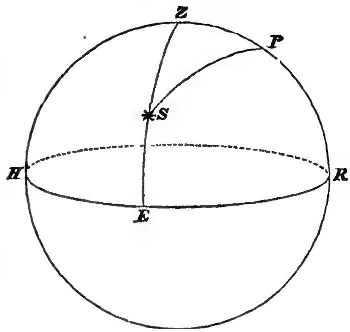
The two bodies should therefore be observed when their azimuths differ by about 90°.

But when the two observations are made with the same instrument, so that the errors of altitude, though unknown, are likely to be the same at both observations, and in the same direction, that is, both in excess, or both in defect, let RV and RV' be two arcs, having S and S' for poles, and for spherical radii the erroneous zenith distances $z - \theta$ and $z' - \theta$ respectively, then R will be the erroneous zenith so determined; and it is obvious that, if the meridian ZP be perpendicular to ZR, the co-latitude given by PR will not sensibly differ from the true one PZ.

Now ZV and ZV' being supposed equal, ZR bisects the angle SZS', therefore *the most favourable condition of observation will be when the verticals of the two bodies are on the same side of the meridian, and equally inclined to the prime vertical*, the difference of azimuths being at the same time near 90° if possible.

152. *Fifth Method.* To determine the latitude by observations made with the transit in the prime vertical (Art. 109).

Let S be the star in the prime vertical ZS, P the pole. Let the instant of the star's crossing the mean wire be noted by the sidereal clock. The difference between this and the right ascension of the star will be the angle SPZ, $= \alpha$ suppose.



Let Δ be the polar distance of the star.

The right-angled triangle SPZ gives $\cos P = \tan PZ \cot PS$.

$$\cot. lat = \cos\alpha \tan\Delta.$$

The nearer the star passes to the zenith, the less will be the effect of an error in the time of transit.

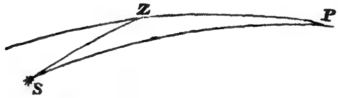
153. If the same star be observed in the prime vertical both on the east and on the west side of the meridian, we shall require neither the right ascension of the star nor the sidereal time, and shall thus be independent of the error of the clock provided its rate be known. The angle α of the formula will be one-half the elapsed sidereal time between the two observations.

When the transit instrument admits of reversion on its bearings, all instrumental errors may be eliminated by observing a star on opposite sides of the meridian on one night; and, after reversing the instrument, again observing the same star on another night; but for a fuller discussion of the method and of the various refinements which render it valuable, we must refer to *Chauvenet's* Spherical Astronomy.

Circum-meridian Observations.

154. *Sixth Method.* To determine the latitude by the altitude of a heavenly body observed very near the meridian.

Let h be the hour angle ZPS of the sun or star;
δ the declination $= 90° - PS$,
ϕ the latitude $= 90° - PZ$,
z the observed zenith distance ZS;
$z - x =$ the meridian zenith distance $= PS - PZ$,
$= \phi - \delta$.



The triangle ZPS gives

$$\cos ZS = \cos PZ \cos PS + \sin PZ \sin PS \cos ZPS$$

$$= \cos(PS - PZ) - 2 \sin PZ \sin PS \sin^2 \tfrac{1}{2} ZPS,$$

$$\cos z = \cos(z - x) - 2 \cos\phi \cos\delta \sin^2 \tfrac{1}{2} h,$$

$$2 \sin(z - \tfrac{1}{2}x) \sin \tfrac{1}{2}x = 2 \cos\phi \cos\delta \sin^2 \tfrac{1}{2} h,$$

$$\sin \frac{x}{2} = \frac{\cos\phi \cos\delta \sin^2 \frac{1}{2} h}{\sin(z - \frac{1}{2}x)},$$

and approximately, since x is small,

$$x = \frac{2 \cos\phi \cos\delta}{\sin z} \sin^2 \frac{h}{2},$$

and $$lat. = z - x + \delta.$$

It will be observed, that the value of x is obtained in terms of ϕ, the unknown latitude, but we may there replace ϕ by its approximate value $z + \delta$.

155. The following modification of the method would be found useful at sea. The observations being made within a short distance from the meridian, h is small; we may therefore write $\frac{1}{2}h$ for $\sin \frac{1}{2}h$, and, if we suppose x expressed in seconds of a degree and h in minutes of time,

$$x \sin 1'' = \frac{2 \cos\phi \cos\delta}{\sin z} \left(\frac{h \sin 15'}{2}\right)^2,$$

$$x = \frac{\cos\phi \cos\delta \sin^2 15'}{2 \sin z \sin 1''} h^2.$$

The multiplier of h^2 may be considered constant, and equal to its meridian value, which we shall call C;

therefore $$C = \frac{\cos\phi \cos\delta \sin^2 15'}{2 \sin(\phi - \delta) \sin 1''},$$

and $$x = C.h^2.$$

C is the change of altitude during the first minute before or after passing the meridian, and, being a function of the latitude and declination, its values may be conveniently tabulated. These values change very slowly, so that we may use that which corresponds to the roughly estimated latitude.*

All that the observer has to do, therefore, is to multiply the square of the number of minutes which elapse between the observation and the meridian passage by the proper factor C, and the result, added to the observed altitude, will give the meridian altitude, whence the correct latitude may be found.

The precision of the method is really much greater than might at first be supposed, especially when the body does not pass near the zenith. Thus, in latitude 50°, when the declination is 0°, the error will not exceed 1′, so long as the hour angle is less than 40^m; it will be less than 30″ when the hour angle does not exceed 33^m; and for about a quarter of an hour on each side of the meridian the error will be less than 1″, and consequently much within the probable errors of observation.

Passing clouds may hide the sun at the very instant when it is exactly on the meridian, and yet allow very good observations to be made on each side. But even when no clouds interfere with the meridian altitude, the mean result of several observations taken near the meridian,

* See Mendoza Rios's *Tables for Navigation and Nautical Astronomy.* See also Lieut. Raper's *Practice of Navigation*, Third Edit., which contains methods analogous to the above.

and corrected by this method,* will be of greater value than that of any single observation, even a meridian one.

156. The method just given supposes the hour angle known with tolerable accuracy. When this is not the case, we may proceed as follows:

Let two observations be made, and the interval t be noted;

Let z, z' be the two observed zenith distances,

x, x' the corrections to reduce them to the meridian,

h, h' the corresponding hour angles,

$$x = Ch^2,$$

$$x' = Ch'^2;$$

therefore $z - z' = x - x' = C(h^2 - h'^2) = C(h - h')(h + h')$;

$z - z'$ is the change of altitude, and one of the two factors $h - h'$ or $h + h'$ will be t, the known interval of time. This equation will, therefore, furnish the other factor,

$$h \mp h' = t,$$

$$h \pm h' = \frac{z - z'}{Ct};$$

therefore $$h = \tfrac{1}{2}\left(t + \frac{z - z'}{Ct}\right),$$

whence $x = Ch^2$; and $z - x$, the meridian zenith distance, will be known.†

157. *Seventh Method.* To determine the latitude by an observation of the pole star.

This method is especially valuable at sea, because the observations may be made at any time when the star is visible; and, on account of the proximity to the pole, the

* Each observation must be corrected separately, for the arithmetical mean of the altitudes will not correspond to the arithmetical mean of the times.

† See Dubois's *Cours de Navigation et d'Hydrographie.* Chauvenet's *Spherical and Practical Astronomy.*

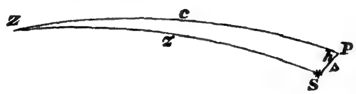
motion is always slow, so that an error in the estimated time has little influence on the result.

Let c be the co-latitude of the place,

h the hour angle of the star,

Δ its polar distance,

z the zenith distance $= 90^\circ - a$, where a is the observed altitude corrected for refraction, &c.



If $c - z = altitude - latitude = x$, then x and Δ will be small quantities.

$$\cos z = \cos c \cos\Delta + \sin c \sin\Delta \cos h$$
$$= \cos(z + x)\cos\Delta + \sin(z + x)\sin\Delta\cos h$$
$$= (\cos z - x\sin z - \tfrac{1}{2}x^2\cos z + \tfrac{1}{6}x^3\sin z + \ldots)(1 - \tfrac{1}{2}\Delta^2 + \ldots)$$
$$+ (\sin z + x\cos z - \tfrac{1}{2}x^2\sin z + \ldots)(\Delta - \tfrac{1}{6}\Delta^3 + \ldots)\cos h.$$

We shall proceed to express x in a series of ascending powers of Δ. Dividing by $\sin z$, and neglecting the fourth and higher powers of small quantities,

$$\cot z = \cot z - x + \Delta\cos h - \tfrac{1}{2}x^2\cot z - \tfrac{1}{2}\Delta^2\cot z + \Delta x\cot z\cos h$$
$$+ \tfrac{1}{6}x^3 + \tfrac{1}{2}x\Delta^2 - \tfrac{1}{2}x^2\Delta\cos h - \tfrac{1}{6}\Delta^3\cos h;$$

therefore $\quad x = \Delta\cos h - \tfrac{1}{2}\cot z(x^2 + \Delta^2 - 2\Delta x\cos h)$

$$+ \tfrac{1}{6}(x^3 + 3x\Delta^2 - 3x^2\Delta\cos h - \Delta^3\cos h);$$

whence $\quad x = \Delta\cos h$ is a first approximation,

and, substituting this value in the terms of the second order,

$x = \Delta\cos h - \tfrac{1}{2}\Delta^2\cot z\sin^2 h$ is a second approximation.

This second approximation being used in the terms of the second order, and the first approximation in those of the third order, we obtain

$$x = \Delta\cos h - \tfrac{1}{2}\Delta^2\cot z\sin^2 h + \tfrac{1}{3}\Delta^3\cos h\sin^2 h;$$

or, if x and Δ be expressed in seconds,

$$x = \Delta\cos h - \tfrac{1}{2}\Delta^2\sin 1''\cot z\sin^2 h + \tfrac{1}{3}\Delta^3\sin^2 1''\cos h\sin^2 h.$$

The maximum value of the last term corresponds to

$$3 \cos^2 h - 1 = 0,$$

and its value is then $\frac{2}{9\sqrt{3}} \Delta^3 \sin^2 1''$. Now Δ is at present less than $1° \, 30'$, and the value of this term is, therefore, less than $0''\cdot5$. We may, therefore, neglect it, and we have

$$lat. = a - \Delta \cos h + \tfrac{1}{2}\Delta^2 \sin 1'' \tan a \sin^2 h.$$

The Nautical Almanac contains tables for facilitating the computation. The first table gives the values of $\Delta \cos h$, the argument* being the sidereal time of observation, which differs from h by a constant quantity. This first correction applied to a gives an approximate latitude $a - \Delta \cos h$.

The second table gives the value of the term

$$\tfrac{1}{2}\Delta^2 \sin 1'' \tan a \sin^2 h,$$

the arguments being the sidereal time and the altitude.

A third table is also given, to allow for the difference between the actual value of Δ and that value which is employed in the construction of the first table. The arguments are the *date* and the sidereal time.

* The *argument* of a table is the known quantity which serves to determine the value of some dependent function.

CHAPTER X.

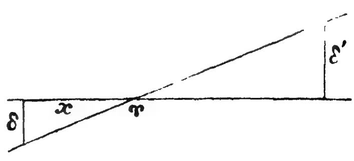
THE ECLIPTIC.

Determination of the First Point of Aries.

158. The first point of aries (♈)—that point of the celestial equator which the sun's centre occupies when crossing from the south to the north side—is the zero point from which all right ascensions are reckoned (Art. 16). On account of its importance, it is necessary that we should know with great accuracy its position among the fixed stars, or, which amounts to the same thing, the position of the stars relatively to it, as expressed by their right ascensions. (See Chap. VII., Arts. 121, 122).

First Method. To determine the instant when the sun is in ♈. Let us find by observation the sun's declination at noon on two successive days (Art. 119), selected so that it may be south on the first and north on the second of the two, having crossed the equator in the interval.

Let δ be the south and δ' the north observed declinations, and let α, β be the corresponding differences of right ascension between the sun and some chosen fundamental fixed star (Art. 120),



then $\delta + \delta'$ the change of declination,

and $\alpha - \beta$ ....right ascension,

may be considered as taking place uniformly during the

interval of twenty-four hours; so that, if x be the change of right ascension between the first position and ♈,

$$\frac{x}{\alpha-\beta}=\frac{\delta}{\delta+\delta'}.$$

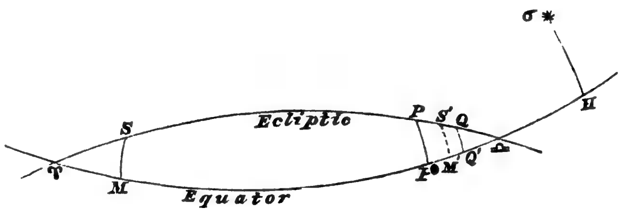
Therefore, the stars's right ascension

$$=\alpha-x=\alpha-\frac{(\alpha-\beta)\,\delta}{\delta+\delta'}$$

$$=\frac{\alpha\delta'+\beta\delta}{\delta+\delta'}.$$

159. *Second Method. Flamsteed's.* Let ♈M♎ be the equator, ♈S♎ the ecliptic, and σ a fixed star.

Let the zenith distance z of the sun be observed at noon on some day near the vernal equinox, and also the



difference α between the time of its transit and that of the star σ, and let δ be the declination of the sun.

Thus, if S be the sun's place at noon, SM and σH the declination circles of the sun and star,

$$SM=\delta,\quad MH=\alpha.$$

Now, when the sun is approaching the autumnal point ♎, let P and Q be two of its positions at noon on successive days, when the zenith distances z' and z'' are—the one less and the other greater than z.

Suppose these to have been observed, and also the corresponding intervals $P'H=\beta$, $Q'H=\gamma$, between the transits of the sun and the same fixed star σ.

We may assume that, during the motion of the sun from P to Q, the changes of declination and of right ascension

proceed at a uniform rate; so that if S' be the sun's position when the declination $S'M'$ equals SM, we have

$$\frac{P'M'}{P'Q'} = \frac{PP' - S'M'}{PP' - QQ'} = \frac{\delta' - \delta}{\delta' - \delta''},$$

or $$P'M' = \frac{\delta' - \delta}{\delta' - \delta''}(\beta - \gamma) = \frac{z - z'}{z'' - z'}(\beta - \gamma);$$

therefore $$MM' = \alpha - \beta + \frac{z - z'}{z'' - z'}(\beta - \gamma),$$

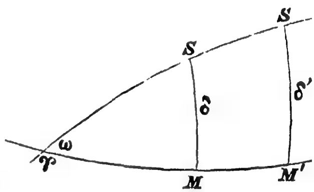
and sun's right ascension at S is

$$\Upsilon M = 90^\circ - \tfrac{1}{2}MM'$$
$$= 90^\circ - \tfrac{1}{2}(\alpha - \beta) - \tfrac{1}{2}\frac{z - z'}{z'' - z'}(\beta - \gamma),$$

star's right ascension

$$= \alpha + \Upsilon M = 90^\circ + \tfrac{1}{2}(\alpha + \beta) - \tfrac{1}{2}\frac{z - z'}{z'' - z'}(\beta - \gamma).$$

160. *Third Method.* Both the right ascension and the inclination ω of the ecliptic to the equator, or the obliquity of the ecliptic, as it is called, may be determined by two observations of the sun's declination and of the change of right ascension in the interval.



Let S, S' be the two positions of the sun on the ecliptic $\Upsilon SS'$; δ, δ' the observed declinations, and

ΥM, or Æ, the sun's right ascension at first observation

$\Upsilon M'$, or Æ $+ \alpha$, ... second .... ,

α being the change of right ascension, determined, as in the previous methods, by comparison of the times of transit of the sun and some star.

The right-angled triangles ΥMS, $\Upsilon M'S'$ give

$$\sin Æ = \cot\omega \tan\delta,$$
$$\sin(Æ + \alpha) = \cot\omega \tan\delta',$$

therefore $$\frac{\sin(\text{Æ} + \alpha)}{\sin \text{Æ}} = \frac{\tan \delta'}{\tan \delta},$$

$$\cos\alpha + \sin\alpha \cot \text{Æ} = \tan\delta' \cot\delta,$$

whence $$\cot \text{Æ} = \tan\delta' \cot\delta \operatorname{cosec}\alpha - \cot\alpha.$$

This determines the sun's right ascension, and from it may at once be found the right ascension of the star. The obliquity will be given by

$$\cot\omega = \sin \text{Æ} \cot\delta.$$

161. This method is exact, but there are practical advantages in Flamsteed's which render it valuable, although only approximate. If we compare the formulæ, we shall see that Flamsteed's does not require the absolute declinations of the sun, but only the changes of declination, or of zenith distance between the first and second, and between the second and third observations: so that any uncertainty in the latitude of the observer, or any instrumental or other errors* which would affect each observation equally, will not influence the result.

The right ascension of a fundamental star being thus found, that of any other celestial body will be obtained by observing the interval between its transit and that of the star.

162. In the chapter on precession, we shall see that ♈ is not a fixed point as we have hitherto supposed it, but that it has a slow—very slow—motion among the stars. There will in consequence be a slow change in their coordinates, determined at different epochs. Tables of the Æ and decl. of 100 of the principal stars are given in the Nautical Almanac for every *tenth* day, and those of the sun for *every* day at Greenwich noon. The rate of change during the intervals being nearly uniform, we may calculate

* Such as uncertain values of parallax or refraction,—because in this method the sun has nearly the same altitude at the different observations.

the Æ or decl. for intermediate times. The Æ is generally tabulated in hours, minutes, and seconds; but it will be easy to convert it, if desired, into degrees at the rate of 15° for every hour.

Determination of the Obliquity of Ecliptic.

163. Perhaps the most accurate method of determining the obliquity (ω) is by observations of the meridian zenith distance of the sun at the two solstices. At these epochs the sun's declination is exactly equal to the obliquity; and if z, z' be the observed zenith distances at summer and winter solstices respectively, and λ the latitude, we have

$$\lambda - \omega = z,$$

$$\lambda + \omega = z';$$

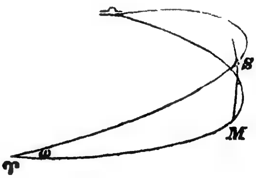
therefore $$\omega = \tfrac{1}{2}(z' - z).$$

The same observations give us also the latitude of the place

$$\lambda = \tfrac{1}{2}(z + z').$$

164. We must remark, however, that the sun is not likely to be exactly 90° from ♈ when he crosses the meridian; because, as he occupies the solstitial point only for an instant, he may at that instant be far from the observer's meridian, if not actually below the horizon.

A correction will be necessary to allow for the small change in declination in the interval.



Let ♈S♎ be the ecliptic,

S the sun near the solstice,

SM the decl. $= \delta = \omega - x$,

♈M the right ascension $= 90^\circ \pm \alpha$,

where x and α are small quantities;

$$\sin(90 \pm \alpha) = \tan\delta \cot\omega,$$

$$\cos\alpha = \frac{\tan\delta}{\tan(\delta + x)},$$

$$\frac{1-\cos\alpha}{1+\cos\alpha} = \frac{\tan(\delta+x)-\tan\delta}{\tan(\delta+x)+\tan\delta},$$

$$\tan^2\frac{\alpha}{2} = \frac{\sin x}{\sin(2\delta+x)},$$

this determines x, and it may be expanded in the form

$$x = \tan^2\frac{\alpha}{2}\sin 2\delta + \tfrac{1}{2}\tan^4\frac{\alpha}{2}\sin 4\delta + \ldots,$$

where the first term will be sufficient when the observations are made within five or six days on either side of the solstice.

Let z be the observed zenith distance and λ the latitude, then if λ is accurately known, $\delta = \lambda - z$ will also be known, and thence $\omega = \delta + x$ may be obtained by *observations near one solstice only.* But any uncertainty in the value of λ will attach to δ and, through it, to ω. The value of x will also be affected, but the error will not be appreciable, and we may consider $z - x$ as accurately known;

therefore $$\lambda - \omega = z - x.$$

Similar observations at, or near, the winter solstice will give

$$\lambda + \omega = z' - x',$$

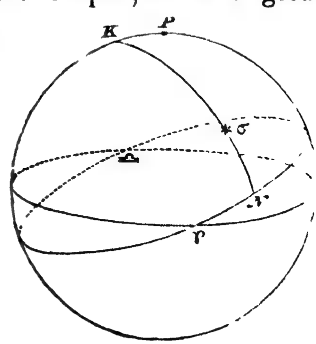
whence $$2\omega = (z' - x') - (z - x),$$

with an accuracy depending on that of z and z'.

Celestial Latitude and Longitude.

165. The position of a celestial body may be referred to the ecliptic instead of the equator.

Thus, if K be the pole of the ecliptic, $K\sigma N$ a great circle through a star σ, meeting the ecliptic in N, the position of the star will be known when ΥN and $N\sigma$ are given; these are called the longitude and latitude respectively.



The *latitude* is the arc of a great circle drawn from the body perpendicular to the ecliptic.

The *longitude* is the portion of the ecliptic intercepted between this circle and the first point of Aries.

166. The latitude takes its name *north* or *south* from that pole of the equator which is on the same side of the ecliptic as the body.

Those stars which are situated in the acute angles, formed by the ecliptic and the equator, will have their latitude and declination of opposite names; that is, one north, the other south; but for all other stars they will be of the same name.

The longitude is measured eastward from ♈ through 360°.

The ancients, who always referred the places of bodies to the ecliptic, subdivided the 360° of longitude into twelve equal parts called *Signs*, and to these they gave the names of the constellations which occupied those signs in the early days of astronomical science. Thus, the first 30° from ♈ was called Aries, the next 30° Taurus, and so on, the names and symbols of the twelve signs being—

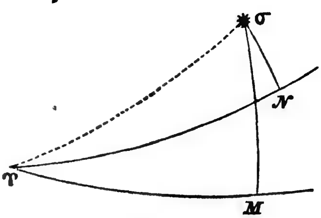
Aries.	Taurus.	Gemini.	Cancer.	Leo.	Virgo.
♈	♉	♊	♋	♌	♍
Libra.	Scorpio.	Sagittarius.	Capricornus.	Aquarius.	Pisces.
♎	♏	♐	♑	♒	♓

These have fallen into disuse, and the constellations have so far shifted, that the appropriateness of the names has been lost. The first sign is no longer in the constellation Aries, but in that of Pisces. This shifting we shall explain in Chap. XVIII.

Transformation of Coordinates.

167. The obliquity of the ecliptic being known, we can readily convert Æ and decl. into latitude and longitude, and *vice versâ*.

Thus, let σ be a star, σM and σN arcs of great circles perpendicular to the equator and ecliptic respectively; join ♈σ by an arc of great circle.



Let ΥM the rt. ascension $= Æ$,

$M\sigma$ ... declination $= \delta$,

ΥN ... longitude $= l$,

$N\sigma$ ... latitude $= \lambda$,

$N\Upsilon M$ ... obliquity $= \omega$.

Firstly. Suppose $Æ$ and δ given, then the right-angled triangle $\sigma\Upsilon M$ gives

$$\cos\Upsilon\sigma = \cos Æ \cos\delta \quad\text{..................}(\alpha),$$

$$\cot\sigma\Upsilon M = \sin Æ \cot\delta \quad\text{..................}(\beta).$$

Therefore in the right-angled triangle $\sigma\Upsilon N$, we know $\Upsilon\sigma$ and $\sigma\Upsilon N = \sigma\Upsilon M - \omega$, thence

$$\sin\lambda = \sin\Upsilon\sigma \sin(\sigma\Upsilon M - \omega) \quad\text{..............}(\gamma),$$

$$\tan l = \tan\Upsilon\sigma \cos(\sigma\Upsilon M - \omega) \quad\text{..............}(\delta),$$

(α) and (β) give the auxiliary quantities $\Upsilon\sigma$ and $\sigma\Upsilon M$, then (γ) and (δ) determine λ and l.*

Secondly. Suppose λ and l given. The steps will obviously be the exact counterpart of those we have just described, thus

$$\cos\Upsilon\sigma = \cos l \cos\lambda \quad\text{..................}(\alpha'),$$

$$\cot\sigma\Upsilon N = \sin l \cot\lambda \quad\text{..................}(\beta')$$

determine the auxiliary quantities $\Upsilon\sigma$ and $\sigma\Upsilon N$, then

$$\sin\delta = \sin\Upsilon\sigma \sin(\sigma\Upsilon N + \omega) \quad\text{............}(\gamma'),$$

$$\tan Æ = \tan\Upsilon\sigma \cos(\sigma\Upsilon N + \omega) \quad\text{............}(\delta'),$$

determine the declination and right ascension.

168. In the case of the sun, whose latitude is zero, the connection between the right ascension, declination, longitude,

* These formulæ may be modified so as to require only one of the auxiliary quantities, viz., the angle $\sigma\Upsilon M$. If we represent it by ϕ, it may be easily shewn that

$$\cot\phi = \sin Æ \cot\delta,$$

$$\sin\lambda = \frac{\sin(\phi - \omega)\sin\delta}{\sin\phi},$$

$$\tan l = \frac{\cos(\phi - \omega)\tan Æ}{\cos\phi}.$$

and a similar modification may be applied to the second set of equations.

and obliquity, is given by the solution of the right-angled triangle $S\Upsilon M$ (fig., p. 132), whence

$$\cos l = \cos Æ \cos \delta,$$

$$\sin \delta = \sin \omega \sin l,$$

$$\tan \delta = \sin Æ \tan \omega,$$

$$\tan Æ = \cos \omega \tan l.$$

When two of the four quantities Æ, δ, l, ω are known, these equations will determine the others.

169. From the last of the equations above, we get

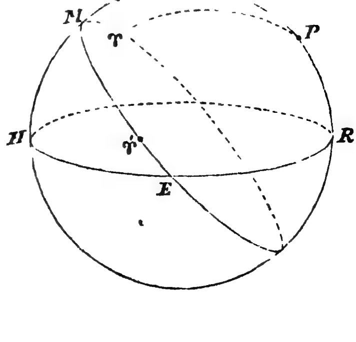
$$\sec^2 Æ \frac{dÆ}{dt} = \cos \omega \sec^2 l \frac{dl}{dt};$$

therefore $$\frac{dÆ}{dt} = \cos \omega \sec^2 \delta \frac{dl}{dt};$$

therefore corresponding small increments of the sun's Æ and longitude will be in the ratio of $\cos \omega : \cos^2 \delta$.

Right Ascension of the Meridian.

170. By the right ascension of the meridian is meant the Æ of those stars which are in the meridian at the instant considered. It is, therefore, the sidereal time expressed in degrees (Art. 17), and is equal either to the hour angle of the first point of Υ, or to its defect from 24^h, according as Υ is on the west, or on the east side of the meridian.



Thus, if σ be a star in the meridian, ΥME the equator, Υ being on the west of the meridian, then ΥM is the Æ of the meridian, and is obviously the same as ΥPM, which is the hour angle of Υ.

But if Υ be on the east side of the meridian as at Υ',

the Æ of the meridian would be the whole circumference *minus* $\Upsilon' M$, that is, the whole circumference *minus* the hour angle of Υ'.

The right ascension of the meridian may also be found as follows:—Take from the Nautical Almanac the sun's Æ for the given time, and add to it the time past noon, deducting 24^h from the result if it exceeds that sum.

Position of the Ecliptic at a Given Instant.

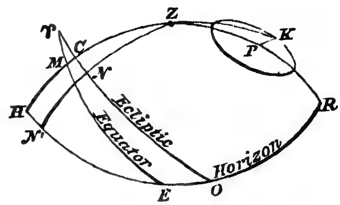
171. If the equator were a bright visible band in the sky, it would occupy a fixed position, of which we may readily obtain an idea by means of some of its points, viz. the east point with its opposite the west point, and a point in the meridian at a distance from the zenith equal to the latitude.

The position of the ecliptic is not so readily conceived, on account of its constant shifting. We must determine the point where it crosses the eastern horizon, called the *ascending point*, which at the same time gives the opposite or *descending point;* these, with some third point or with the inclination of the ecliptic plane to the horizon, will be sufficient.

When the sun is above the horizon, his centre, which is always in the ecliptic, will give us the third point we want; or, we may find the point called the *culminating point*, where the meridian is cut by the ecliptic; or, again, we may find the *nonagesimal*, which is 90° from the ascending point, and is the highest point of the ecliptic above the horizon.

172. Let the ecliptic, equator, and horizon be as represented in the figure on the next page, the meridian being ZCM. Let ΥM be the right ascension of the meridian (calculated as in Art. 170), which we shall call M.

Let Z be the zenith, P the pole, K the pole of the ecliptic. Let KZ meet the ecliptic in N and the horizon in N', then N is the nonagesimal, and NN' its altitude is equal to the inclination of the ecliptic to the horizon.



Let ZM, the latitude of the place, $=\phi$.

In the triangle ΥEO, we know

$$\Upsilon E = 90^\circ + M,$$

$$\angle E = MR = 90^\circ + \phi,$$

$$\angle \Upsilon = \text{obliquity of the ecliptic} = \omega,$$

whence EO and the angle O can be found.

$$\cot EO \sin \Upsilon E = \cot \Upsilon \sin E + \cos \Upsilon E \cos E;$$

therefore $\cot EO = \cot \omega \cos \phi \sec M + \tan M \sin \phi \ \ldots\ldots (\alpha),$

$$\cos O = -\cos \Upsilon \cos E + \sin \Upsilon \sin E \cos \Upsilon E;$$

therefore $\cos O = \cos \omega \sin \phi - \sin \omega \cos \phi \sin M \ \ldots\ldots (\beta),$

(α) gives the ascending point, and (β) the inclination or its equivalent, the altitude of the nonagesimal, and thus the position of the ecliptic is determined.

173. If C be the culminating point of the ecliptic, we have from the right-angled triangle ΥMC,

$$\tan MC = \sin M \tan \omega,$$

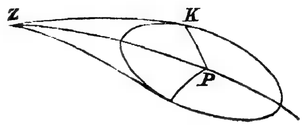
and $$ZC = \phi - MC,$$

whence the culminating point is known, and this, with the ascending point given by (α), will also fix the position of the ecliptic.

174. E and O being the poles of ZP and ZK respectively, EO is equal to the angle PZK.

In the course of twenty-four hours K describes a small circle round P, and the angle PZK varies between certain

limits—the greatest value corresponding to the position when ZK is a tangent to the small circle, and consequently the angle at K a right angle.



Then

$$\sin PZK = \frac{\sin PK}{\sin PZ} = \frac{\sin \omega}{\cos \phi}.$$

Hence, the greatest value of EO is $\sin^{-1}\left(\frac{\sin \omega}{\cos \phi}\right)$, and the ascending point oscillates to this extent from one side to the other of the east point; but the time from the most northerly to the most southerly position will not be the same as the time back again; for the angle ZPK is acute, and the ratio of the two times will be $\frac{\pi - ZPK}{ZPK}$.

When K is in the meridian above the pole, ♈ rises, and the three points ♈, O, and E all coincide; also KZ has its least value $= 90 - \phi - \omega$, which is therefore the least inclination of the ecliptic to the horizon.

When K is in the meridian below the pole, ♎ rises at E, and the angle between the ecliptic and the horizon has then its greatest value $= 90 - \phi + \omega$.

At a place within the frigid zone the result will be somewhat different. There, Z will fall within the small circle described by K, and the point O will travel completely round the horizon, the ascending suddenly becoming the descending point, and *vice versâ*. The greatest and least values of the inclination will be $\omega + (90 - \phi)$ and $\omega - (90 - \phi)$.

175. The longitude of the nonagesimal is an element sometimes wanted in the calculation of eclipses. Its value is easily inferred from the triangle ♈EO, for ♈N = ♈$O - 90°$,

$$\cot ♈O \sin ♈E = \cot E \sin ♈ + \cos ♈E \cos ♈;$$

therefore $\tan ♈N = \tan \phi \sin \omega \sec M + \tan M \cos \omega$.

CHAPTER XI.

FORM OF THE EARTH'S ORBIT.

176. THE orbit of the sun round the earth, or of the earth round the sun, lies, as we have seen, in one plane; but what particular curve is described in this plane we have not yet ascertained. We shall proceed to an examination of this question, which, until the time of Kepler, remained without a satisfactory solution.

It is very easily perceived, that the distance of the earth from the sun is not a constant quantity. For, by taking accurate measurements of his apparent diameter at different times,* we find that it varies from 31′ 31″·0 on the 1st of July to 32′ 35″·6 on the 31st of December, and as we cannot suppose the magnitude of the sun to vary periodically, we must infer that his distance changes, and that the earth's orbit is not a circle with the sun at the centre. The angles subtended by the sun being small, must be very nearly inversely proportional to the distances.

Again, if the angular motion of the sun in his orbit be observed, that is, his angular motion in longitude, its daily value will be found to vary from 0° 57′ 11″ 5 to 1° 1′ 9″·9, these minimum and maximum values occurring at the same epochs of July 1st and December 31st.

177. In the first place we shall remark, that the change in angular velocity is greater than that in apparent diameter;

* An instrument called a *Heliometer* is specially adapted to this purpose. It is a telescope with a divided object-glass on the principle of Dollond's double-image micrometer (Art. 112) See Chauvenet's *Astronomy*, vol. II.

for, the change of the former is about $\frac{1}{15}$ of its mean value, that of the latter only $\frac{1}{30}$. Or, while

the apparent diameter changes in the ratio of $1 : 1 + \frac{1}{30}$,

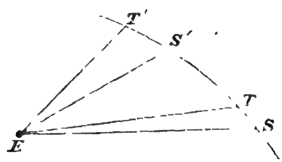
the angular velocity .... $1 : 1 + \frac{1}{15}$,

which latter is the same as $1 : (1 + \frac{1}{30})^2$ nearly,

and, at whatever times the numerical values be compared, it will always be found that

$$\text{angular velocity} \propto (\text{appt. diam.})^2,$$

$$\propto \frac{1}{(\text{dist.})^2}.$$



Now let E be the earth, S the sun at any time, and ST the arc described in one day.

Let $S'T'$ be the arc also described in one day at any other time.

Then sector SET : sector $S'ET'$

$= \angle SET \times (SE)^2 : \angle S'ET' \times (S'E)^2$

$=$ angular vel. at $S \times (\text{dist.})^2$: angular vel. at $S' \times (\text{dist.})^2$,

therefore the sector $SET =$ sector $S'ET'$.

Or, *the areas, described in equal times by a radius vector joining the earth and sun, are equal in all parts of the orbit.*

This is one of Kepler's famous laws discovered after long and laborious calculations. This law tells us how the orbit is described, but says nothing about its form.

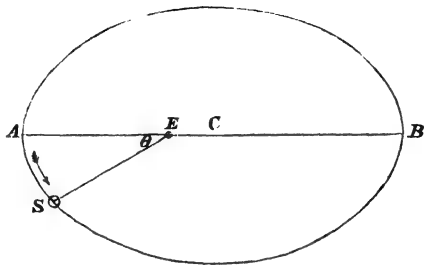
178. Another of Kepler's laws, also the result of many years' study and observation, is, that the orbit is an ellipse, of which the earth occupies one focus, and this we may verify in the following manner:

Let the apparent diameter of the sun, and his corresponding angular distance from the position of the 31st of December, that is, his change of longitude, be observed day by day throughout the year.

Let E be the earth,

A the sun's place on the 31st of December,

B .... 1st of July,



when it has described 180° of longitude.

$$AE : EB = \frac{1}{32' \, 35''\cdot 6} : \frac{1}{31' \, 31''\cdot 0},$$

$$AE : AB = 31' \, 31'' : 64' \, 6''\cdot 6,$$

and if $$AB = 2a, \quad AC = a,$$

we find $$AE = 0\cdot 98321a,$$

$$EB = 1\cdot 01679a;$$

therefore $$EC = \cdot 01679a.$$

Then, if an ellipse be described with AB for major axis, and E for one focus, any radius vector ES, making with AE an angle θ, will be numerically calculated from the formula

$$ES = \frac{a(1-e^2)}{1+e\cos\theta}, \quad \text{where } e = \cdot 01679;$$

therefore $$\frac{1}{ES} \propto 1 + e\cos\theta,$$

hence, if the sun describes this ellipse, we ought to find

$$\text{apparent diam.} \propto 1 + \cdot 01679 \cos AES,$$

a relation which is verified by the observations. Therefore the sun describes an ellipse about the earth in one focus, or, according to the more correct statement, *the earth describes an ellipse about the sun in one of the foci* (Art. 133).

179. The points of the orbit where the sun is at his greatest and least distances are called respectively *Apogee* and *Perigee*, when, to conform our language to appearances, we attribute the motion to the sun; but, when we employ the more correct supposition and regard the earth as the body moving round the sun in the focus, the corresponding points of the earth's orbit are called *Aphelion* and *Perihelion*, and the line joining them the *line of Apsides.*

180. The laws of elliptic motion having been thus established by observation, it will now be easy to explain why the seasons should be of different lengths, as stated in Art. 135. Let us make the earth revolve about the sun.

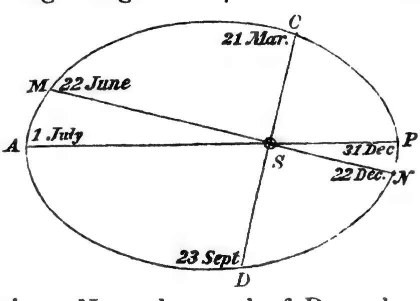
Let S be the sun in the focus of the orbit,

AP the apse line.

The earth will be at P on the 31st of December, and at A on the 1st of July.

Let CD be the line of equinoxes,

MN at right angles to it, the line of solstices.



The earth is at N on the 22nd of December, only 9 days before reaching P.

A simple inspection of the figure shews that the winter quadrant NSC, which contains the least radius vector SP, must be the smallest, and the opposite summer quadrant MSD, the greatest. Of the two others, the spring quadrant CSM is larger than the autumn one DSN. The areas of the quadrants being unequal, so also will the times of describing them be.

181. At present the lengths of the seasons are as given in Art. 135. We say—at present—because any change in the position of AP relatively to CD will obviously affect these lengths; and our observations will show that both the line of equinoxes and the line of apsides are in motion, and in opposite directions; so that the angle PSC is annually diminishing by about $61''\cdot47$. When, in about 2100 years, the line SP bisects the angle NSC, summer and winter will have respectively their greatest and least possible values, and spring and autumn will be of equal lengths. When, after 2650 years longer, SP coincides with SC, summer and autumn will be equal, and longer than winter and spring, which will also be equal to one another. The subsequent changes can be easily followed.

Of the $61''\cdot47$ of annual change in the angle PSC, $50''\cdot22$ are due to a retrograde motion of SC (see Chap. XVIII.). The remaining $11''\cdot25$ are due to a progressive motion of the Apse-line, which we proceed to determine.

To determine the Position and Motion of the Apse-line.

182. We may remark that when the sun is in perigee, its distance from the earth is least, and consequently its apparent diameter greatest; and then also it attains its greatest angular velocity. But the changes in these quantities are so slow for several days before and after, that it is impossible to detect, by direct observation, the precise instant when this occurs. It may, however, be done with tolerable accuracy by determining points on each side of perigee or apogee, at which either the diameters or the angular velocities are equal; then the apse line will bisect the angle formed by the radii drawn to these positions.

183. A more accurate method of determining the instant and the position of perigee consists in observing the interval of time between two positions of the sun separated by 180°.

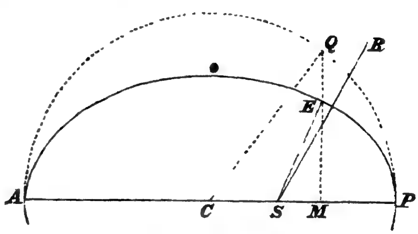
When this interval is exactly half the time required to describe 360°, the two positions must be perigee and apogee, since no other line through the focus but the apse line bisects the area.*

By a comparison of the position of the apse line at present with those which it has occupied at different times separated by long intervals, it is found to have a progressive motion of 11″·25 a year.

Position of the Earth in its Orbit at any Time.

184. When the instant of perihelion is known, and also the time required to make a complete revolution in the orbit, it is clear that there must be some means of ascertaining the position of the body corresponding to a given time. This we proceed to explain :

Let S be the sun at the focus, P the perihelion, E the earth at time t, measured from perihelion, PSE being the angle described.



We have already said that the radius vector joining the sun and earth does not revolve at a uniform rate. Let us conceive a uniformly revolving radius SR to start with the actual radius SE from perihelion, and let the angular velocity of SR be such that the two radii accomplish their revolution in exactly the same time; then they will coincide at perihelion and at aphelion, and there only. The angular velocity of SR is called the *mean angular velocity* of the earth.

* For other methods of determining the position of the line of apsides see Delambre's *Astronomie*, vol. II. p. 158.

The angle PSE, described by the true radius from perihelion, is called the *true anomaly;* the angle PSR, described in the same time by the uniformly revolving radius, is called the *mean anomaly;* and the angle ESR between them is the *equation of the centre.* Therefore

True anom. = *Mean anom.* + *Equation of centre.*

There is yet another angle connected with the position of E which we shall find it useful to consider. It is the angle QCP at the centre of the ellipse, formed by the apse line CP and the line joining C with Q, the point of the auxiliary circle which corresponds to E. This is called the *excentric anomaly.*

To find the Relation between the Mean *and the* Excentric *Anomalies.*

185. Let n be the *mean* angular velocity of the earth, so that if T be the periodic time

$$n = \frac{2\pi}{T}.$$

Then, mean anomaly at time $t = PSR = nt$.

And, if we suppose SQ joined, we have, by a property of the ellipse, and by Kepler's law,

$$\frac{\text{area of } PSQ}{\text{area of circle}} = \frac{\text{area of } PSE}{\text{area of ellipse}} = \frac{t}{T} = \frac{nt}{2\pi};$$

but, $\quad$ area of PSQ = sector PCQ − triangle SCQ

$$= \tfrac{1}{2}a^2u - \tfrac{1}{2}a^2e \sin u,$$

where u is the excentric anomaly, a the semi-major axis, and e the excentricity; therefore

$$nt = u - e \sin u \quad \text{.......................(i).}$$

To find the Relation between the True *and the* Excentric *Anomalies.*

186. Referring to the figure, we have, θ being the true anomaly,

$$SE \cos\theta = CM - CS;$$

therefore $$\frac{a(1-e^2)}{1+e\cos\theta}\cos\theta = a\cos u - ae,$$

$$(1-e^2)\cos\theta = (\cos u - e)(1+e\cos\theta);$$

whence $$\cos\theta = \frac{\cos u - e}{1 - e\cos u},$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{(1+e)(1-\cos u)}{(1-e)(1+\cos u)},$$

$$\tan\frac{\theta}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)}\tan\frac{u}{2} \quad \text{..............(ii).}$$

187. The excentric anomaly is thus, by means of (i) and (ii), a link between the mean and the true anomalies. When θ is given, t can be readily found; but the more usual problem is the inverse of this, and requires us to find the value of θ corresponding to a given value of t. This might be done by employing a succession of trials to approximate to the value of u in (i) and then determining θ by (ii). But, the quantity e being small, the plan adopted is to express u and thence θ in a series of ascending powers of e, by Lagrange's Theorem (see *Differential Calculus*), and retain only the important terms. We shall refer for the investigation to Tait and Steele's *Dynamics* (Chap. VI.). The result to the third power of e is

$$\theta = nt + 2e\sin nt + \tfrac{5}{4}e^2\sin 2nt + \frac{e^3}{12}(13\sin 3nt - 3\sin nt),$$

and $$\textit{Equation of centre} = 2e\sin nt + \tfrac{5}{4}e^2\sin 2nt + \&c.$$

When θ has been found, the value of $\dfrac{SE}{a} = \dfrac{1-e^2}{1+e\cos\theta}$ may be determined: or it may also be expressed, in terms of the time, in a series of ascending powers of e,

$$\frac{SE}{a} = 1 - e\cos nt + \frac{e^2}{2}(1-\cos 2nt) + \frac{3e^3}{8}(\cos nt - \cos 3nt.$$

188. If the longitude of the apse be added to the true anomaly, we get the true longitude of the sun; and if added

to the mean anomaly we get what is called the mean longitude, therefore

True long. = Mean long. + Equation of centre.

189. Besides the two laws, given in this chapter, which were found by Kepler to hold in the case of each planet, he, a few years later, discovered a *third law*, which establishes a remarkable connection between the periodic times of the different planets. This law, which furnishes one of the arguments for the earth's being a planet (Art. 133, (2)), is that "*The squares of the periodic times vary as the cubes of the semi-major axes.*"

Kepler's three laws, and the other results of this chapter, can be shewn to be necessary consequences of the law of universal attraction—Newton's great discovery; but that method of investigation belongs to Physical Astronomy, whereas we are concerned with Astronomy considered as a science of observation.

CHAPTER XII.

UNITS OF TIME. EQUATION OF TIME.

190. FOR astronomical purposes the *sidereal day* is one of the principal units of time. It begins at the instant when the first point of aries is on the meridian; a correct sidereal clock should then mark $0^h\ 0^m\ 0^s$, and at any other instant the *sidereal time* will be the hour angle of ♈ reckoned westward from 0^h to 24^h (Art. 17).

A *solar day* is the interval between two successive transits of the sun's centre over the meridian. The sun changes his right ascension, advancing eastward among the stars, at the rate of about 1° a day; therefore the earth will have to turn nearly 361° about its axis to complete a solar day, which will consequently be about 4^m longer than a sidereal day.

The solar time at any instant is the hour angle of the sun's centre reckoned westward from 0^h to 24^h. This is called the *apparent solar time*, and is the time indicated by a sun-dial.

If the sun's motion in right ascension were uniform, the solar days would be all equal to one another, but this is not the case. In the first place, the sun's motion in its own orbit is not uniform; and secondly, even if it were, the corresponding motion in right ascension would not be uniform on account of the inclination of the orbit to the equator.

191. The solar day, marking the recurrence of light and darkness, is obviously that on which man, in civil life, must regulate his time, although the want of uniformity mentioned above hinders us from employing it as a measuring unit. We may, however, obtain a uniform measure of time

depending on the sun in the following manner. Conceive an imaginary body, called the *mean sun*, to move along the equator with the mean angular velocity of the true sun. The days marked by this mean sun will be all equal, and exactly the average of all the solar days during the year. Therefore a clock, whose motion is necessarily uniform, may be regulated on the mean sun. To complete the connection between the two suns, we must establish the starting point of the mean sun; and it will be convenient so to choose it that the mean solar time and the apparent solar time may never be widely separated. The following has been adopted: Conceive another imaginary body, say a star, to have the same uniform angular velocity as the mean sun, but to move along the ecliptic instead of the equator, and to pass through perigee at the same time as the true sun; then the motion of the mean sun is so adjusted that it may pass through ♈ at the same time as the star.

By referring to Art. 188, it will be seen that the connection between the two suns may be expressed by saying that *the right ascension of the mean sun is equal to the mean longitude of the true sun:* because the mean longitude of the true sun is the longitude of the supposed star.

192. *Mean noon* is the instant when the mean sun is on the meridian; and the *mean time* at any instant is the hour angle of the mean sun reckoned westward from 0^h to 24^h. These twenty-four hours constitute the astronomical mean day; but for civil purposes it is found more convenient to begin the day at midnight and complete it at the next midnight, dividing it into two periods of 12 hours each. The two reckonings only agree in the afternoon of each day, thus

Aug. 26 at 9 P.M. civil time is Aug. 26 at 9^h astro. time,
but Aug. 27 at 9 A.M. .... Aug. 26 at 21^h ....

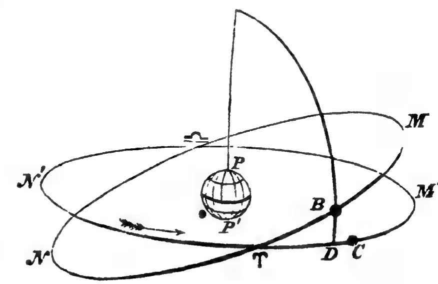
193. The *equation of time* is the difference between apparent and mean time at any instant; it is usually considered

as the correction to be applied to the former to obtain the latter, and is therefore called positive when mean noon precedes true noon, and *vice versâ.*

It is obvious that the equation of time is the value, expressed in time, of the angle between the declination circles of the true and the mean suns.

In order to examine the variation in the equation of time, we may consider separately the two causes to which it is due, and the algebraic sum of the two effects will be approximately that due to their joint action.

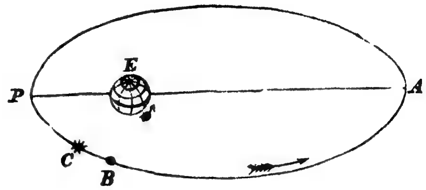
194. *Firstly.* Neglecting the elliptic motion, let us suppose the sun to describe his orbit ♈M ♎ N with uniform angular velocity, the mean sun describing the equator ♈M' ♎ N' with the same velocity, the two passing through ♈ at the same instant, and the earth at the centre turning about its axis PP' in the same direction (marked by the arrow) once a day.



When the true sun is at B, the mean sun will be at C, where ♈C = ♈B, and, if the declination circle BD be drawn through B, CD will measure the angle between the declination circles of B and C, and therefore the equation of time.

It is obvious that C and D will coincide only at the equinoxes and solstices. From equinox to solstice, C will be in advance of D, and behind it from solstice to equinox. Hence, while the sun is between ♈ and M, any given meridian will, as the earth revolves, overtake first the true sun B and then the mean sun C, *i.e.* apparent noon will precede mean noon, and the equation of time will be subtractive. In the same way it may be shewn to be additive from solstice to equinox.

195. *Secondly.* Neglecting the obliquity of the ecliptic, let PBA be the sun's elliptic path, B the place of the true sun between perigee and apogee, C the corresponding place of the star or mean sun, the two coinciding at perigee and apogee only.



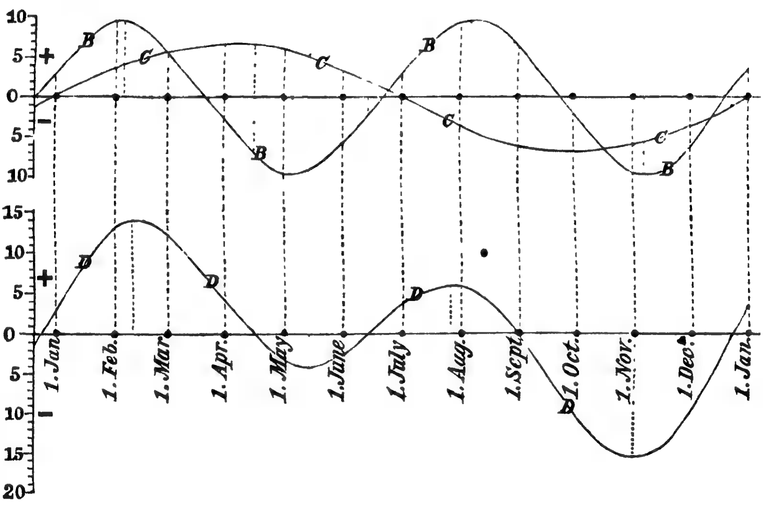
At perigee the true sun has its greatest velocity, and will, therefore, shoot ahead of the mean, the interval between them continuing to increase so long as the sun's angular velocity exceeds its mean value. This will be at somewhat more than 90° from perigee (Art. 198); after this the interval will begin to diminish and the two will again coincide at apogee. Therefore, from perigee to apogee the successive meridians of the earth will overtake the mean sun before the true, *i.e.* mean noon will take place before true noon, or the equation of time will be additive.

A similar reasoning will show that from apogee to perigee the equation of time will be subtractive.

This gives merely the general character of the equation as due to this cause; its actual value will be obtained by multiplying the angular interval by $\cos\omega \sec^2\delta$, which is the factor requisite to reduce a small arc from the ecliptic to its corresponding projection on the equator (Art. 169). Since the multiplier varies with the declination, it follows that the greatest equation of time, due to this cause, will not necessarily correspond to the greatest equation of the centre. Moreover, the position of the apse line with respect to the first point of aries is (Art. 181) changing by about $61''{\cdot}47$ annually, and therefore the same equations of the centre will not recur with exactly the same declinations, and this also will slowly affect the maximum equation of time due to the ellipticity of the orbit.

196. It remains now to combine the two parts of the equation of time into one sum, and when this is done we find that it vanishes four times a year, on or about the following dates:—April 15th, June 15th, August 31st, and December 24th. The following graphical method of effecting the combination will perhaps shew the changes, and speak to the eye, more forcibly than tabulated numbers alone would do:

Draw a horizontal line—the upper one in the figure—to represent the time, the successive days being supposed represented by equal successive intervals.



Let the curve *BBBB* be drawn, so that its ordinates, corresponding to each day, may represent that part of the equation of time, on that day, which is due to the obliquity of the ecliptic, positive values being set off on one side of the line and negative values on the other; the curve will cross the straight line at the equinoxes and solstices, the maximum value at intermediate times being about 10^m (see Art. 197).

Again, draw the curve $CCCC$ to represent in a similar manner that part of the equation of time due to the elliptic form of the orbit, and whose greatest value is now about 7^m (see Art. 198). This will cross the straight line on the 31st of December and on the 1st of July.

To avoid confusion, draw another horizontal line—the lower one in the figure—equal and parallel to the former, to represent times as before; and let the curve $DDDD$ be traced by making its ordinates equal to the algebraic sum of the ordinates of the two former curves; this curve will represent the equation of time due to the combination. The figure shews the four vanishing positions specified above, and likewise the intermediate maximum values, viz.:

+ 14·5 min. about February 11th,
− 3·9 .... May 14th,
+ 6·2 .... July 25th,
− 16·3 .... November 1st.

197. If we refer to Art. 169, we have

$$\frac{d\text{Æ}}{dt} = \cos\omega \, . \sec^2\delta \, \frac{dl}{dt};$$

but, when $l - \text{Æ}$ is a maximum, $\dfrac{dl}{dt} = \dfrac{d\text{Æ}}{dt}$;

therefore
$$\cos\omega \, . \sec^2\delta = 1,$$
$$\cos\delta = \sqrt{(\cos\omega)},$$

which value of δ will, by the formulæ of Art. 168, lead to

$$\sin(l - \text{Æ}) = \tan^2\frac{\omega}{2},$$

and therefore the maximum value of this part of the equation of time $= l - \text{Æ} = 2° \, 28' = 10$ min. in time nearly. The separate values of l and Æ will be found 46° 14′ and 43° 46′ from the equations

$$\sin l = \cos \text{Æ} = \frac{1}{\sqrt{(2)}} \sec\frac{\omega}{2}.$$

198. Again, the equations in Art. 187 will enable us to obtain the maximum value of the equation of the centre; and this, when powers of e above the second are neglected, is found $= 2e$ nearly, the corresponding value of θ being about 90°. But $e = \frac{1}{60}$ (Art. 178); therefore maximum equation of the centre $= \frac{1}{30} = 1° \, 55'$ nearly; and, as this takes place at present near the equinoctial point where $\delta = 0$, the factor $\cos\omega . \sec^2\delta$ becomes $\cos 23° \, 27\frac{1}{2}'$, or $\frac{11}{12}$ nearly, and the corresponding part of the equation of time will be $\frac{11}{12}$ of $1° \, 55' = 1° \, 45' = 7$ min. in time.

The equation of time should also take account of other disturbances, such as—those due to precession, which produce unequal changes in the longitude and right ascension of the sun (Chap. XVIII.). But for this and for the effect of planetary disturbances we shall refer to Delambre's *Astronomie*, vol. II.

Equinoctial Time.

199. In addition to the three kinds of time—sidereal, apparent or solar, and mean solar—there is another, sometimes used by astronomers, and called equinoctial time, which has the advantage of being independent of the observer's meridian. When a phenomenon is observed at different places, the time of its occurrence will be registered differently, although the absolute instant may be the same for all. This, as we know, is due to differences of longitude; noon being reckoned, and the astronomical day beginning, at each place, at the moment the mean sun crosses the meridian of that particular place. Therefore, in order to institute a comparison between the observations, it is necessary to specify the place as well as the time of observation.

The *equinoctial time* is the time, expressed in mean days, that has elapsed since the preceding mean vernal equinox.

The equinoctial time is therefore reckoned from an epoch which is common to all observers.

Different kinds of Years.

200. A year is the period of the earth's revolution about the sun, from some determinate position back again to the same.

If the starting point be a star or some point fixed among the stars, the interval is called a *sidereal year*.

If we start from the first point of ♈, which, as we have already stated, has a retrograde motion of $50''\cdot22$ annually, moving as it were to meet the earth, the period will not be so long. This is called the *tropical year*, and, as it determines the commencement of the seasons and all the important phenomena of vegetation and life, it is the unit marked out by nature for the use of man. By observations, separated by a long interval, it is found to consist of 365·242216 mean solar days.

A third year is obtained by taking for our starting point the perihelion of the earth's orbit. Observation shews that the apse line has a progressive annual motion of $11''\cdot25$ (Art. 183); the earth will therefore have to move through this quantity, in addition to the 360°, in order to complete this period, which is called the *anomalistic year*.

The relative magnitudes of the three years will therefore be

$$\frac{\text{tropical}}{360^\circ - 50''\cdot22} = \frac{\text{sidereal}}{360^\circ} = \frac{\text{anomalistic}}{360^\circ + 11''\cdot25},$$

and the tropical year having been found 365·242216 days,

the sidereal year will be .... 365·256374 ......,

the anomalistic year will be .... 365·259544 ....

The *civil year* contains an exact number of days, either 365 or 366. Its adjustment and dependence on the tropical year will be explained hereafter (Chap. XXIII.).

CHAPTER XIII.

REDUCTION AND CONVERSION OF TIME. FINDING THE TIME BY OBSERVATION.

201. As the earth turns uniformly on its axis, one meridian after the other is brought opposite to the sun, and the different places have their noons in succession, according to their longitude (Art. 31). The solar time at a given place being the angle made by the sun's declination circle with the meridian of that place, it follows that the difference between the solar times at two different places, at the same instant, will be exactly the angle between the meridians of those two places; that is, their difference of longitude. The same will be true of the mean solar times or of the sidereal times; and, generally, the difference of longitude will be equal to the difference of the hour angles of any, the same, celestial point at the same instant.

202. Therefore, "to find the time under any meridian corresponding to a given time at some other meridian," we must convert the difference of longitude into time, at the rate of 15° per hour, and add to, or subtract from, the given time: bearing in mind that, as the earth turns from west to east, the more easterly meridian will have its noon first, and must therefore reckon a more advanced time.

For example, the longitude of the Paris observatory is 2° 20′ 9″·45 east; that of the observatory at Pulkowa is 30° 19′ 39″·9 east; what is the mean time at Paris when it is $1^h\ 5^m\ 12^s$ of September 3rd at Pulkowa?

			h. m. s.
Long. of Pulkowa	30° 19′ 39″·9 E.	Mean time at Pulkowa, Sept. 3rd	1 5 12
" Paris	2° 20′ 9″·45 E.		
Diff of Longitude	27° 59′ 30″·45	= in time	1 51 58 03

to be subtracted, Pulkowa being the more easterly; therefore mean time at Paris ....Sept. 2nd 23 13 13 97

203. A curious consequence of this difference of local times will be the gain or loss of a day to a person travelling right round the world.

We see from the above example that, supposing it possible for a person starting from Paris at noon to reach Pulkowa the next day at the noon of Pulkowa, he would have completed a day according to his own estimation, the sun having returned to his meridian, whereas the inhabitants of Paris would only reckon $22^h\ 8^m$ to have elapsed; the traveller would thus have gained $1^h\ 52^m$ on those he left behind, and if he continued to travel eastward, it is easy to see that this gain would go on increasing.

The general explanation of the gain or loss of a day may be easily given as follows: Every time that a person is carried completely round the axis of the earth, relatively to the sun, he reckons one day to have elapsed. Therefore, supposing him to start from any place and to travel eastward until he returns to his starting-point, whatever number of turns the earth may make in the interval, that is, whatever number of days may be reckoned by those persons who have remained stationary, he will have made one turn more by his own motion, and therefore reckon one day more. If, on the contrary, he travel westward, or in the direction opposite to the earth's rotation, he will, as it were, cancel one of the turns which the earth has made, and reckon one day less.

For a similar reason, two ships starting from England, and meeting, say in New Zealand, the one having gone round Cape Horn, the other round the Cape of Good Hope,

will also differ in their reckoning by one day. The practical inconvenience of using two different days in the same place would oblige the settlers there to select between the two; and it seems that in New Zealand, Australia, and Polynesia generally, that day has prevailed which was brought round the Cape of Good Hope, probably from the earliest settlers having gone by that route.

Ships usually change their reckoning at some point in the Pacific between New Zealand and America.

Equivalent Sidereal and Mean Solar Intervals.

204. The sun advances among the stars in the same direction—west to east—as the earth revolves about its axis; any given meridian, therefore, in the course of a tropical year, crosses the first point of aries exactly once oftener than it does the sun. Now, the number of mean solar days in a tropical year is 365·242216, Art. 200, therefore the number of sidereal days in the same time is 366·242216. Hence, if M and S be the measures of the same interval expressed in mean and sidereal times respectively, we have

$$\frac{M}{365\cdot242216} = \frac{\text{given time}}{\text{one year}} = \frac{S}{366\cdot242216},$$

whence,

$$\frac{M}{S} = \frac{365\cdot242216}{366\cdot242216} = 1 - k,$$

$$\frac{S}{M} = \frac{366\cdot242216}{365\cdot242216} = 1 + k',$$

$$k = \cdot00273043, \quad k' = \cdot00273791.$$

To facilitate the reduction, the Nautical Almanac contains tables which give the values of M corresponding to any number of hours, minutes, seconds, tenths, and hundredths of S, and conversely.

In some tables the values of kS and $k'M$ are registered opposite to the values of S and M respectively, then $S - kS$ gives M, or $M + k'M$ gives S.

One sidereal day contains $23^h\ 56^m\ 4^s{\cdot}0906$ of mean time,
one mean solar day .... $24^h\ 3^m\ 56^s{\cdot}5554$ of sidereal time.

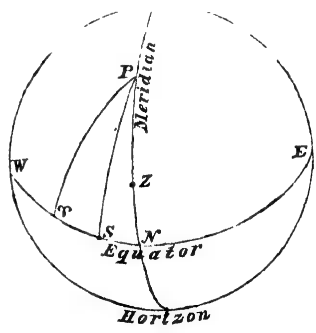
Conversion of Time.

205. 1°. *To convert the apparent solar time at a given place into mean solar time*, and conversely.

Here the equation of time only is required. The Nautical Almanac gives this equation for the apparent noon, and also for the mean noon of each day at Greenwich, on pages I. and II. of the successive months. If then, by applying the longitude as in Art. 202, we obtain the Greenwich time corresponding to the given local time, and, according as it is apparent or mean time which is given, refer to the tables in page I. or page II. of the month, and make the necessary correction for the change since noon, we shall obtain the equation of time: this equation, applied with its proper sign to the local time, will convert it from apparent to mean, or from mean to apparent.

206. 2°. *To convert the mean solar time at a given meridian into the corresponding sidereal time.*

Let PZN be the given meridian, S the mean sun, and ♈ the first point of aries.



Then, NPS, the hour angle of the mean sun, reckoned westward, is the mean time; NP♈, the hour angle of ♈, is the sidereal time; and ♈PS is the right ascension of the mean sun. Hence

sid. time = mean time + mean sun's right ascension ... (i).

Now the value of the mean sun's right ascension is registered for the instant of Greenwich mean noon, in page II. of each month in the Nautical Almanac, under the heading of "Sidereal Time." This we shall denote by $Æ_0$.

We must, therefore, determine the Greenwich mean time corresponding to the given local time, by Art. 202, and correct the value of Æ$_0$, for the change in the interval, by multiplying this interval by the factor k' of Art. 204, that is, by allowing a uniform change at the rate of $3^m\ 56^s{\cdot}555^s$ in 24 hours, or $9^s{\cdot}8565$ in 1 hour. The corrected right ascension, added to the local mean time, will, according to the above formula, give the sidereal time.

If M be the mean time at the given place;

L .... *west* longitude of the place in time;

Æ$_0$ .... mean sun's right ascension at previous Greenwich noon, from page II. of Nautical Almanac,

then $M + L =$ mean time at Greenwich,

Æ$_0 + k'(M + L) =$ mean sun's right ascension;

therefore, by (i),

sidereal time, or S, $= M +$ Æ$_0 + k'(M + L)$ .... (ii).

Ex. At Madras, in longitude 80° 14′ 19″·5 east, an observation is made on September 6th, 1865, at $9^h\ 21^m\ 12{\cdot}8^s$ mean time; find the corresponding sidereal time.

	h.	m.	s.
Mean time at Madras, September 6th	9	21	12·8
Longitude in time, *east*	−5	20	57·3
Mean time at Greenwich, September 6th	4	0	15·5

	h.	m.	s.
Æ$_0$ or sidereal time at noon, Sept. 6th, page II. of Nautical Almanac	11	2	21·45
Change in $4^h\ 0^m\ 15^s{\cdot}5$	+	0	39·47
Mean sun's Æ	11	3	0·92
Mean time at Madras	9	21	12·8
Sidereal time	20	24	13·72

207. 3°. *To convert the sidereal time at any meridian into the corresponding mean time.*

The solution of this problem is at once given by equation (ii) of the last article. From it we obtain

$$M = \frac{S - Æ_0 - k'L}{1 + k'}$$

$$= (1 - k)(S - Æ_0) - kL, \text{ because } (1 + k') = \frac{1}{1 - k}$$

$$= S - Æ_0 - k(S - Æ_0 + L);$$

where it must be remembered that L is positive in west, and negative in east longitude.

The factor k produces a change of $3^m\ 55^s{\cdot}9094$ in 24 hours, or $9^s{\cdot}8296$ per hour.

Ex. Suppose the sidereal time at Madras, September 6th, 1865, to be $20^h\ 24^m\ 13{\cdot}72^s$, find the corresponding mean time.

	h.	m.	s.	h.	m.	s.
$S=$	20	24	13·72			
$Æ_0=$	11	2	21·45			
$S - Æ_0=$	9	21	52·27	= 9	21	52·27
$L=$	− 5	20	57·3			
$S - Æ_0 + L=$	4	0	54·97			
which multiplied by k, gives				−	0	39·47
therefore mean time				9	21	12·8

When a star is in the meridian, its right ascension is equal to the sidereal time;—the above rule will therefore enable us to determine the mean time of transit of a known star.

208. 4°. *To convert the apparent time at any meridian into the corresponding sidereal time*, and conversely.

Following the rules of the preceding articles, we may convert the apparent time into mean time, and then the mean time into sidereal time; or inversely.

Or, we may proceed directly by using the formulæ of Arts. 206 and 207, taking M to represent apparent time, provided that, instead of $Æ_0$ we employ the apparent right ascension of the sun at apparent noon, as given in page I. of the month in the Nautical Almanac; although the change

for one hour is no longer constant, its value, as given in the Almanac, may be considered constant during that day.

Finding the Time by Observation.

209. First method. *By meridian observations.*

In a fixed observatory, fitted with a transit instrument, the readiest and most accurate way of finding the time is by noting the time of transit of a known star. By observing it at the different wires, and making the corrections and reductions explained in Chap. IV., Art. 79—83, the exact instant marked by the clock when the star is in the meridian will be determined, and the known right ascension of the star will be the corresponding sidereal time, which may be reduced to mean time if necessary (Art. 207), and thus the *error* of the clock on sidereal or mean time will be known.

If the clock error be found in this manner at two different dates, separated by a few days, the change in the error divided by the number of days elapsed will give the average *daily rate* during that interval. With a good clock this daily rate will remain uniform for a considerable time.

Having thus found the error and rate of a clock, *i.e.* how much it was too fast or too slow on a given day, and how much it gains or loses daily, we may easily find the true time at any instant corresponding to a given clock time.

210. When the sun is observed, if the times of transit of both limbs be noted, the mean of these will be the time of transit of the centre, *i.e.* apparent noon; but if only one limb be observed, allowance must be made for the time (given in page I. of the month in the Nautical Almanac) that the semi-diameter takes to cross the meridian.

This will determine the error of the clock on apparent time, and thence the error on mean time, by applying the equation of time.

211. Second Method. *By a single altitude of a known star, or of the sun, moon, or planet.*

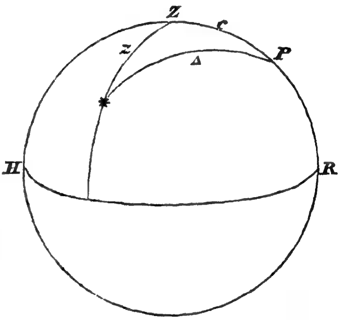
When the body is at some distance from the meridian, let its altitude be observed with a sextant or alt-azimuth instrument, and let the corresponding clock-time be noted.

In the case of the sun, moon, or planet, the altitude observed is necessarily that of the lower or upper limb, and the semi-diameter (given in the Naut. Alm.) must be added or subtracted to get that of the centre. A star has no appreciable semi-diameter.

Corrections must also be made for any known instrumental errors; for refraction (Chap. XV.); for dip when the observation is made at sea with a sextant (Art. 8); and finally for parallax (Chap. XVI.).

In the case of a star there is no parallax correction.

If a be the corrected alt.,
z the zenith dist. $=90^\circ-a$,
δ the decl^n (from Naut. Alm.)
Δ the polar dist. $=90^\circ-\delta$,
ϕ the latitude,
c the co. lat. $=90^\circ-\phi$,
h the hour angle of the star,



z, Δ, and c being the three sides of the triangle SPZ, the value of h will be given by

$$\sin\frac{h}{2}=\sqrt{\left\{\frac{\sin\frac{1}{2}(z+c-\Delta)\sin\frac{1}{2}(z+\Delta-c)}{\sin c\sin\Delta}\right\}}$$

$$=\sqrt{\left\{\frac{\cos\frac{1}{2}(\phi+\Delta+a)\sin\frac{1}{2}(\phi+\Delta-a)}{\cos\phi\sin\Delta}\right\}}$$

$$=\sqrt{\left\{\frac{\cos s\sin(s-a)}{\cos\phi\sin\Delta}\right\}},\text{ where } s=\tfrac{1}{2}(\phi+\Delta+a).$$

When the body observed is the moon, a planet, or a star, the Naut. Alm. will give its right ascension Æ, then Æ $\pm\frac{h}{15}$

will be the *sidereal time* of observation, the upper or lower sign being used according as the star is west or east of the meridian.

If the sun be the body observed, then the *apparent time* will be $\frac{h}{15}$, if on the west side, or 24 hours $-\frac{h}{15}$, if on the east side, of the meridian.

Instead of a single altitude it will be best to observe a series of altitudes in quick succession, and consider the mean of the altitudes as corresponding to the mean of the times. If the altitudes do not change at a uniform rate, calculate the error of the clock corresponding to each altitude, and take the mean of the errors for the clock error.

212. The more rapid the change of altitude, the less will an error in the observation affect the time deduced from it; let us therefore examine for what position of the body this change is quickest.

We have $\cos z = \cos c \cos \Delta + \sin c \sin \Delta \cos h$,

therefore, h and z being the variables,

$$\sin z \frac{dz}{dh} = \sin c \sin \Delta \sin h,$$

$$\frac{dz}{dh} = \frac{\sin c \sin \Delta \sin h}{\sin z} = \sin c \sin A,$$

where A is the azimuth PZS.

Therefore the altitude changes most rapidly when A is a right angle, *i.e.*, when the object is on the prime vertical; and the nearer the body is to the prime vertical the more favourable will be the position for determining the time, provided, however, that the altitude of the object be not less than 8° or 10°, for otherwise, the uncertainty of the atmospheric refractions so near the horizon would more than counterbalance the previous advantage.

213. Third Method. *By two altitudes of the sun and the elapsed time, or by simultaneous observations of two stars.*

This method will be available when the latitude is unknown. Proceeding, as in Art. 148, we shall determine the latitude and the angle PSZ. This angle, with the co-latitude PZ, and the zenith distance ZS, will determine the hour angle SPZ,

$$\sin \text{hour-angle} = \frac{\sin ZS \sin PSZ}{\sin PZ};$$

and from the hour angle we deduce the time, as in the second method.

214. Fourth Method. *By equal altitudes.*

If the times T and T', marked by a clock, be noted when a star has the same altitude before and after crossing the meridian, then $\frac{1}{2}(T+T')$ will be the time of its meridian transit, and the error of the clock on sidereal time, or on mean time, may be found as in the first method.

One of the advantages of this method is, that any error of graduation of the sextant, or other instrument with which the observations are made, will have no effect on the result, because the two altitudes are taken at the same graduation.

By taking several pairs of observations, the mean result will be probably freed from other accidental errors.

215. If, instead of a star, we employ the sun, a slight correction will be required for the change of declination.

Let z be the common zenith distance of the two observations,

ϕ ...... latitude of the place,

Δ ...... polar distance at apparent noon,

θ ...... small horary *decrease* of polar distance,

T ...... half-sum, or mean, of the two clock-times,

$2t$ ...... elapsed time between the observations,

x ...... small correction to be subtracted from T to obtain the clock-time of apparent noon,

therefore

$t - x$ is the hour angle before noon at first observation,
$t + x$ .... after .... second ....,
$\Delta + t\theta$ ...... polar distance at first observation, neglecting $x\theta$,
$\Delta - t\theta$ .... second ....,

z and ϕ are the same at both observations, and if h be the hour angle corresponding to a polar distance Δ', we have

$$\cos z = \sin\phi \cos\Delta' + \cos\phi \sin\Delta' \cos h,$$

therefore differentiating,

$$0 = (\sin\phi \sin\Delta' - \cos\phi \cos\Delta' \cos h)\, d\Delta' + \cos\phi \sin\Delta' \sin h dh,$$

but $$d\Delta' = -2t\theta,\ dh = 2x;$$

whence $$x = \left(\frac{\tan\phi}{\sin h} - \frac{\cot\Delta'}{\tan h}\right) t\theta.$$

θ being expressed in seconds of arc, x will be so too, but dividing it by 15 will reduce it to seconds of time. We may also write t for h and Δ for Δ', and the clock-time of apparent noon will be approximately

$$T - \left(\frac{\tan\phi}{\sin t} - \frac{\cot\Delta}{\tan t}\right) \frac{t\theta}{15}.$$

The corresponding mean or sidereal time may then be calculated, and the error of the clock on either of them determined.*

The correction x is called the *equation of equal altitudes.*

It must be remembered that θ is the hourly *decrease* of polar distance, and therefore when the polar distance is increasing, θ becomes negative.

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216. At sea, the first method is not applicable; the second, by a single altitude, is that in general use; but the fourth method, by equal altitudes, may be employed advan-

* See Delambre's *Astronomy*, vol. I., p. 559, where the value of the quantities we have here neglected is shewn to be extremely small. See also Chauvenet's *Astronomy*, which contains tables for facilitating the computation of x.

tageously, provided a correction be made for the change of the ship's place during the interval. This correction may be made in the same manner as in Art. 150, where the altitude of the body at the first station is reduced to what it would have been if made at the same instant at the second place; but, as the altitudes would no longer be equal, a further correction becomes necessary, and this is most simply effected by making, at the second place, a couple of observations in quick succession, so as to find the rate of change of the altitude, and thence the time when the altitude is the same as the reduced altitude of the first station. The error of the watch is thus found on the local time of the second station.

In the case of the sun, this may be done without moving the index of the instrument—and therefore without interfering with the principal feature of the method—by observing the times when the lower and upper limbs respectively attain the given altitude; the time the sun takes to move through an altitude equal to his diameter thus becomes known.

Another method consists in finding the equation of equal altitudes and the error of the clock, as if there were no change of place, and then determining the correction of the time due to the change. The error of the clock found in this manner is the error on the local time at the meridian midway between the two stations. For details see Chauvenet's *Astr.*, vol. I., p. 220, and Raper's *Navigation.*

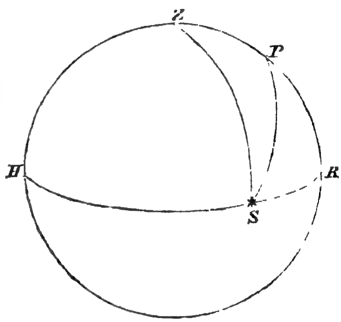
There are various other ways of determining the time by observation, but the preceding are the only ones in constant use. For an account of other methods the reader is referred to Delambre's *Astronomy.*

CHAPTER XIV.

VARIOUS PROBLEMS CONNECTED WITH THE DIURNAL MOTION.

Time of rising or setting of a known body.

217. THIS is really only a particular case of finding the time by a single altitude (Art. 211), but, on account of its simplicity, we shall give a separate investigation.



HSR being the horizon, Z the zenith, P the pole, and S the body, the right-angled triangle PSR gives

$$\cos SPR = \cot SP \tan PR;$$

therefore

$$\cos(\text{hour angle } SPZ) = -\tan(\text{decl.}) \tan(\text{lat.});$$

and the hour angle being known, the time, apparent or sidereal, may be found as in Art. 211.

The hour angle of setting will obviously be the same as that of rising.

218. We have here neglected the refraction which, as we shall see in the next chapter, has the effect of making objects appear higher than they really are, especially near the horizon, where its value amounts to more than 36′; so that, when seen in the horizon, the body is really below it by that amount.

We have also neglected the dip (Art. 8), which still further increases the error, by depressing the visible horizon. The value of the dip, depending on the height of the eye, is variable.

When we wish to take these into account, we must treat the problem as in Art. 211, taking for zenith distance $90° +$ refraction $+$ dip. In the case of the sun, moon, and planets, there will be a further correction for parallax, Chap. XVI., which, acting in the opposite direction, has to be subtracted. The hour angle thus found will be that of the visible or *apparent rising or setting.*

The problem, however, is not one of much practical value, on account of the uncertainty of the refraction near the horizon; and it will generally be sufficient to consider the above solution of it, which determines what is called the *true rising or setting.*

The true rising of the sun occurs when his lower limb is rather more than half his diameter above the visible horizon.

In the case of the moon, the parallax so much exceeds the refraction, that a contrary effect is produced, and the true rising has already taken place even before the upper limb appears.

As to the stars, their light is absorbed by the atmosphere, and they do not become visible until they have attained an altitude of from 5° to 10° above the horizon.

219. *To find the time occupied by the sun in rising at a given place on a given day.*

The sun's rising is accelerated by refraction, &c., but the times of rising of the upper limb and of the lower limb will be accelerated by nearly the same quantity, and the number of seconds occupied by the sun in its visible rising may be calculated with respect to the rational horizon.

If D be the diameter of the sun in seconds, h the hour

angle when the zenith distance is z, δ and ϕ the declination and latitude, n the number of seconds in the interval,

$$\cos z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos h,$$

$$\sin z\, dz = \cos\phi \cos\delta \sin h\, dh,$$

and $z = 90°$ nearly, $\cos z = 0$, $\sin z = 1$; therefore

$$\cos h = -\tan\phi \tan\delta,$$

$$dh = \frac{dz}{\cos\phi \cos\delta \sin h},$$

or
$$15n = \frac{D}{\cos\phi \cos\delta \sqrt{(1 - \tan^2\phi \tan^2\delta)}};$$

therefore
$$n = \frac{\frac{1}{15}D}{\sqrt{\{\cos(\phi+\delta)\cos(\phi-\delta)\}}}$$

determines the number of seconds the sun takes to rise.

Lengths of day and night.

220. If h, or $15t$, be the hour angle of sun-rise or sun-set, δ the declination, and ϕ the latitude, then (Art. 217),

$$\cos h = -\tan\delta \tan\phi,$$

and $2t = \dfrac{2h}{15}$ will be the length of the day in hours,

$2\,(12 - t) = 2\left(\dfrac{180 - h}{15}\right)$..................... night..........

221. If the change of declination during the day be taken into account, the morning and the afternoon will be of different lengths: thus, suppose the declination between sun-rise and sun-set to change from δ to $\delta + \theta''$, and the hour angle from h to $h + dh$,

$$\cos h = -\tan\delta \tan\phi;$$

therefore
$$\sin h\, dh = \sec^2\delta \tan\phi\, d\delta, \text{ approximately,}$$

or
$$dh = \frac{\sec^2\delta \tan\phi}{\sin h}\,\theta = \frac{\sec^2\delta \tan\phi}{\sqrt{(1 - \tan^2\delta \tan^2\phi)}}\,\theta$$

$$= \frac{\sec\delta \sin\phi}{\sqrt{\{\cos(\phi+\delta)\cos(\phi-\delta)\}}}\,\theta.$$

When θ is positive, *i.e.* from winter solstice to summer solstice, the afternoon will be longer than the morning by $\frac{\sec\delta\sin\phi}{\sqrt{\{\cos(\phi+\delta)\cos(\phi\mp\delta)\}}}\frac{\theta}{15}$ seconds of time, and shorter by that quantity during the remainder of the year.

222. If the equation $\cos h = -\tan\delta\tan\phi$ be discussed to determine the length of the day at different places and at different times, we shall find results in exact accordance with the statements of Art. 123...129.

Thus, at a place on the equator, $\phi=0$, therefore $\cos h = 0$, and $h=90°$. Therefore $2t=12$ hours for all values of δ, or the days are always equal to the nights.

At the time of the equinox, $\delta=0$, therefore $\cos h=0$ for all values of ϕ; and the day is then equal to the night all over the earth.

When $\delta = 90-\phi$, $\cos h = -1$, $h=180°$, and the day is 24 hours long.

When $\delta = -(90-\phi)$, $\cos h = 1$, $h=0$, and the sun does not rise.

When $\delta > 90-\phi$, h is imaginary, and the sun neither rises nor sets, but remains entirely above the horizon; and so on for other cases.

223. *To find the hour angle of a body when it has its greatest altitude.*

If the declination be constant, the greatest altitude will be when the body is in the meridian; but when the declination increases, the fall, immediately after passing the meridian, will be more than counter-balanced by the rise due to the increase of declination, and a still greater altitude than the meridian altitude will be attained. If the declination decrease, the greatest altitude will precede the meridian passage.

Let δ be the meridian declination,

α its hourly increase in seconds of arc from Naut. Alm.

θ the circular measure of increase in one second of time,

β the circular measure of 15″,

whence $$\alpha = 60^2 \times 15 \frac{\theta}{\beta}.$$

Let z be the zenith dist. at t seconds past meridian,

ϕ .... latitude,

$$\cos z = \sin(\delta + \theta t) \sin\phi + \cos(\delta + \theta t) \cos\phi \cos\beta t;$$

when z is least, $$\frac{dz}{dt} = 0,$$

$$\left.\begin{array}{r}\theta \{\cos(\delta + \theta t) \sin\phi - \sin(\delta + \theta t) \cos\phi \cos\beta t\} \\ - \beta \cos(\delta + \theta t) \cos\phi \sin\beta t\end{array}\right\} = 0.$$

Expanding, and retaining the important terms,

$$\theta \{\cos\delta \sin\phi - \sin\delta \cos\phi\} - \beta^2 t \cos\delta \cos\phi = 0,$$

$$t = \frac{\theta}{\beta} \cdot \frac{1}{\beta} (\tan\phi - \tan\delta)$$

$$= \frac{\alpha}{60^2 \times 15} \cdot \frac{180 \times 60^2}{15\pi} (\tan\phi - \tan\delta)$$

$$= \frac{4\alpha}{5\pi} (\tan\phi - \tan\delta).$$

In the temperate and in the torrid zones this will always be a small quantity, except in the case of the moon, whose declination sometimes changes rapidly.

Twilight.

224. When the sun disappears below the horizon, darkness does not come on instantaneously, because the rays of the sun, though not coming to us directly, still illumine the atmosphere above us; and the light, reflected and scattered in all directions by the particles of vapour, &c., which are held in suspension, reaches us with an intensity which gradually diminishes as the sun sinks lower.

Observation has shewn that some portion of this diffused light is brought to the observer so long as the sun is not more than 18° below his horizon; after that, darkness begins. A corresponding period also precedes sunrise. The subdued light which thus separates night from day is known as *twilight.*

225. The duration of twilight will vary with the latitude and with the declination. Within the tropics, twilight is always short; because the sun's diurnal path is nearly vertical, and the 18° of depression after sunset are soon attained. At the equator, an interval of 72 minutes separates daylight from complete darkness; but the impression conveyed is that night follows day almost immediately. In high latitudes, the sun's path is so inclined to the horizon that a long interval elapses after sunset before the depression reaches 18°; and at midsummer, in all latitudes exceeding $48\frac{1}{2}$°, it will not, even at midnight, be so much as 18° below the horizon, and there will be no real night.

226. To find the duration of twilight is, therefore, to find the time the sun takes to alter his zenith distance from 90° to 108° in the evening, or from 108° to 90° in the morning.

With the ordinary notation,

$$\cos 108^\circ = \sin\delta \sin\phi + \cos\delta \cos\phi \cos h,$$

determines h the hour angle of the end of twilight, and

$$\cos h' = -\tan\delta \tan\phi,$$

gives h' the hour angle of sunset; whence $h - h'$ the duration of twilight will be known.

227. If $\delta > 72^\circ - \phi$, then $90^\circ - \delta < \phi + 18^\circ$, *i.e.* the polar distance of the sun < latitude + 18°; the sun at midnight will be less than 18° below the horizon, and there will be no real night.

Thus at Cambridge, in latitude 52° 13′, there will be no night while the declination exceeds $72° - 52°\ 13' = 19°\ 47'$ north, or from about the 19th of May to the 24th of July.

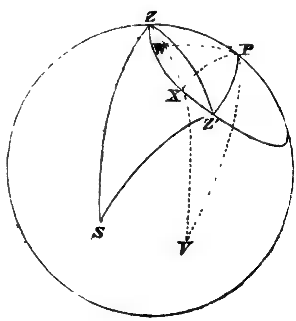
228. *To find the time of the year when twilight is shortest at a given place.*

Let Z be the zenith, P the pole, S the sun at the commencement of morning twilight, when $ZS = 108°$.

Instead of giving the diurnal motion to the sun, let us make the earth revolve; and, when twilight ends, let Z have come to Z', where $Z'S = 90°$; then the meridian PZ will have described the angle ZPZ', which will measure the duration of twilight.

If ZZ' be joined by an arc of a great circle, a triangle ZSZ' will be formed, of which two sides are respectively 90° and 108°, and therefore the third side ZZ' cannot be less than 18°. If, on any day, the three points Z, Z', and S prove to be on the same great circle, ZZ' will be exactly 18°, and twilight will be shortest.

Let Z, X, and V be their positions on that day. Join PV, and draw PW at right angles to ZX.



$PZ = PX = 90° - \phi$; $PV = 90° - \delta$;

$WZ = WX = 9°$; $WV = 99°$.

The right-angled triangles PWV, PWZ, give

$$\cos PV = \cos PW \cos WV,$$

or $$\sin\delta = \cos PW \cos 99°;$$

$$\cos PZ = \cos PW \cos ZW,$$

or $$\sin\phi = \cos PW \cos 9°;$$

therefore $$\sin\delta = -\tan 9° \sin\phi,$$

which determines δ, and thence the time of the year when twilight is shortest.

Also $$\sin WZ = \sin PZ \sin ZPW,$$

or $$\sin 9^\circ = \cos\phi \sin\tfrac{1}{2}ZPX,$$
determines ZPX, the duration of shortest twilight.

Azimuths.

229. The determination of the sun's azimuth has always been of great importance at sea as a means of determining the error of the compass, but the introduction of iron ships has made it a problem of almost daily recurrence. These ships introduce new disturbing causes, and complicate the indications of the compass needle, by combining their own influence with that of the earth, in such a manner that the deviation of the compass not only changes as we pass from one place to another, and from one ship to another, but even in the same ship and at the same place the amount of deviation will often change by many degrees with the direction of the ship's head (see Raper's *Navigation*, and also *Admiralty Manual*, by Com. F. J. Evans, R.N., F.R.S., and A. Smith, Esq., F.R.S.).

If the sun's compass bearing be observed, and his true bearing calculated, the difference will be the correction or error of the compass. This error is the algebraical sum of the *variation of the compass*, or that part which is due to the earth's action, and of the *deviation* which is the part due to the accidental position of the compass with respect to neighbouring masses of iron, &c.

230. *To find the azimuth of the sun at sun-rise or sun-set.*

In the triangle SPR (fig. p. 171), right-angled at R, $SR = PZS$ is the azimuth (Art. 13, note),

$$\cos SP = \cos SR \cos PR, \text{ or } \sin\delta = \cos A \cos\phi;$$

therefore $$\cos A = \frac{\sin\delta}{\cos\phi},$$

which determines A, the azimuth measured from the north in north latitudes, and from the south in south latitudes.

This is the azimuth at the true rising, that is, the azimuth which the sun has when his lower limb is about midway between his centre and the visible horizon (Art. 218).

231. *To find the sun's azimuth by observation of his altitude.* With the usual notation in the triangle SPZ,

$$\cos SP = \cos ZS \cos ZP + \sin ZS \sin ZP \cos SZP,$$

$$\sin\delta = \cos z \sin\phi + \sin z \cos\phi \cos A,$$

$$\cos A = \frac{\sin\delta - \cos z \sin\phi}{\sin z \cos\phi},$$

or $$\sin\frac{A}{2} = \sqrt{\left\{\frac{\cos\frac{1}{2}(\phi+z+\delta)\sin\frac{1}{2}(\phi+z-\delta)}{\cos\phi\sin z}\right\}}.$$

232. *To find the sun's azimuth at a given time of a given day.*

The time reduced to apparent time determines the hour angle SPZ, or h,

then $$\cot PS \sin PZ = \cot PZS \sin SPZ + \cos PZ \cos SPZ,$$

whence $$\cot A = \frac{\tan\delta\cos\phi - \sin\phi\cos h}{\sin h},$$

which may be adapted to logarithms by assuming

$$\tan x = \cot\delta\cos h,$$

then $$\cot A = \frac{\cot h\cos(x+\phi)}{\sin x}.$$

This method has the advantage of being free from errors of atmospheric refraction, and also of being available on many occasions when the altitudes cannot be observed on account of the indistinctness of the horizon.

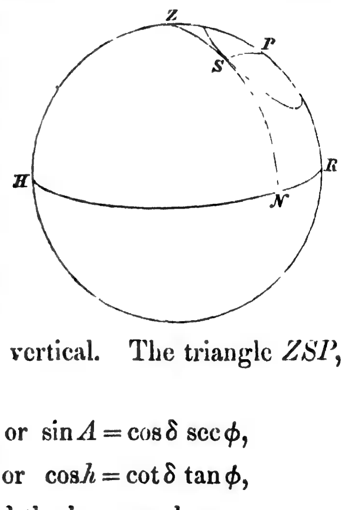
In actual practice at sea, it is sufficient to calculate to the nearest $\frac{1}{2}$ degree, because the azimuth by compass, with which we have to compare the calculated azimuth, can scarcely be ascertained to within a degree.

On account of the importance of the problem, the author will probably be excused for referring here to his method

of solving it by means of a diagram, which he has called the *Time-Azimuth-Diagram.* This has been engraved by the Hydrographic Office, Admiralty, and is now used extensively at sea. It gives the azimuth—without calculation—to within one-eighth of a degree, in much less time than calculation would require, and with scarcely a possibility of error, as the operation is of the simplest character and has no variety of cases.

233. *To find the azimuth and the hour angle of a star when its motion is vertical*, the declination being greater than the latitude of the observer.

Since the declination is greater than the latitude, the polar distance will be less than the co-latitude; therefore the small circle described by the star will pass between the zenith and the pole. If, then, a vertical *ZSN* be drawn to touch the diurnal circle of the star, the point of contact *S* will be the position of the star at the moment when its motion is vertical. The triangle *ZSP*, right-angled at *S*, gives



$$\sin PS = \sin PZ \sin PZS, \text{ or } \sin A = \cos\delta \sec\phi,$$

$$\cos P = \cot PZ \tan PS, \text{ or } \cos h = \cot\delta \tan\phi,$$

which determine the azimuth and the hour angle.

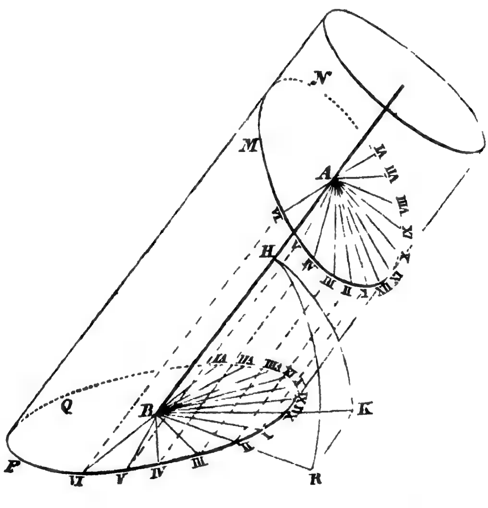
There will be a similar position on the other side of the meridian.

Sun-Dials.

234. The general explanation of the principle of dialing will be easily understood from the following construction, the idea of which is taken from Ferguson's *Lectures*, Edinb. 1806:

Conceive a transparent cylinder, having an axis *AB* parallel to the axis of the earth. On the surface of the cylinder let equidistant generating lines be traced 15° apart, one of them XII–XII being in the meridian plane through *AB*, and the others I–I; II–II, &c., following in the order of the sun's motion.

Then the shadow of the line *AB* will obviously fall on the line XII–XII at apparent noon, on the line I–I at one hour after noon, on II–II at two hours after noon, and so on. If now the cylinder be cut by any plane *MN* representing the plane on which the dial is to be traced, the shadow of *AB* will be intercepted by this plane, and fall on the lines *A*XII, *A*I, *A*II, &c.



The construction of the dial consists in determining the angles made by *A*I, *A*II, &c. with *A*XII; the line *A*XII itself being in the vertical plane through *AB* may be supposed known. Supposing a sphere to be described about *A* as centre, there will always be sufficient data afforded by the position of the plane, and the latitude of the place to enable us to solve the problem.

Thus, if we wish to construct a dial at B on the horizontal plane PQ. Firstly, determine the XII o'clock line BK, and let a sphere described about B cut the axis in H, and the n o'clock line in R.

The triangle HKR is right-angled at K, HK is the elevation of the pole or latitude of the place $= \phi$, KHR (the angle between the planes through the axis of the cylinder, and the lines XII–XII, n–n, respectively) $= n\,15^\circ$, and KR is the required angle θ between BXII and Bn,

$$\sin HK = \tan KR \cot KHR\,;$$

therefore $$\tan\theta = \sin\phi\,\tan n 15^\circ,$$

where, giving to n the values 1, 2, 3, &c., in succession, we shall obtain the angles which the I o'clock, II o'clock, &c. lines make on the plane of the dial with the XII o'clock line.

It scarcely requires to be stated, that the shadow is cast by a rod or style which occupies the position of the axis of the cylinder.

For other positions of the plane, the investigation will proceed on similar principles, but there is no need to dwell on a problem which has lost nearly all its interest and value since the perfection and cheapness of clocks and watches have brought them into general use as measurers of time.

CHAPTER XV.

ASTRONOMICAL REFRACTION.

235. WE have, on several occasions in the foregoing chapters, had to refer to refraction; and we have always supposed that our observations were freed from the errors arising from it, in those cases where it affected the observed values. We shall now proceed to examine what refraction is, and what corrections are necessary on account of it.

It is a well-known principle of optics that a ray of light moves through a transparent medium in a straight line so long as the density of the medium remains uniform; but that in passing obliquely from one such medium to another its course will be bent at the point of incidence in such a manner as to satisfy the following conditions: the two directions before and after incidence will be in one plane with the normal to the surface at that point; the angles formed with the normal will have their sines in a constant ratio so long as the media remain the same.

When the ray passes from vacuum into a medium, the constant ratio of the sines is called the coefficient of refraction for that medium; and when the ray passes from one medium into another, it is easily shewn that the sines of the angles will be inversely as the coefficients of refraction.

Thus, if A and B be the two media,

μ_a ...... μ_b ... their coefficients of refraction,

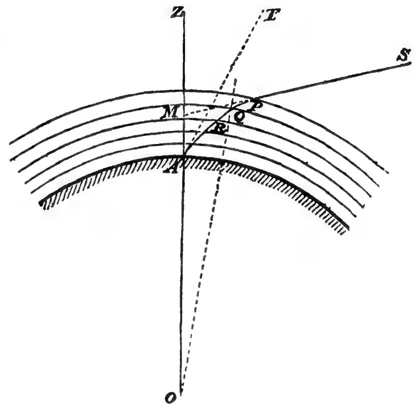
α ...... β ... the angles made with the normal,

then
$$\frac{\sin\alpha}{\sin\beta} = \frac{\mu_b}{\mu_a}.$$

Now, the atmosphere which surrounds the earth may be considered as formed of a series of concentric layers, the

density being uniform throughout each layer, but diminishing rapidly as we recede from the surface. Any plane through the centre of the earth and through a star will intersect these layers in concentric circles, and radii drawn in this plane will all be normals to the surfaces of the layers.

By the preceding laws, therefore, a ray $SPQR\ldots$ which commences its path in this plane will, on account of the increasing density near the surface, describe a broken line or curve concave to the centre of the earth. Let A be the point where it meets the surface. To an observer there, the star will appear in the direction of the ray when it reaches the eye, that is, in the direction AT of the tangent to the path at A.



The *astronomical refraction* is the angle between this apparent direction and that in which the star would be seen if there were no atmosphere.

If SP, produced, meet the vertical AZ in M, SMZ will be the true zenith distance of the star, and TAZ the apparent zenith distance.*

Refraction, therefore, diminishes the zenith distances of all celestial bodies, but does not affect their azimuths.

General Differential Equation.

236. If ϕ, ϕ' be the angles made by two consecutive elements PQ, QR of the path with the normal OQ, whose

* In strictness, we ought to join A with the star, to obtain the true zenith distance; but, on account of the immense distance of all heavenly bodies, the line so drawn would be sensibly parallel to MS.

length, measured from the centre, is x; μ, μ' the coefficients of refraction of the two strata, and p, p' the perpendiculars from the centre on the two directions, then

$$\mu : \mu' :: \sin\phi' : \sin\phi$$

$$:: \frac{p'}{x} : \frac{p}{x};$$

therefore $\quad \mu p = \mu' p' = \mu'' p'' = \ldots = \text{constant},$

a result which is independent of the thickness of the strata; and, when the path becomes a curve, *the perpendicular on the tangent at any point, multiplied by the coefficient of refraction at that point, is constant throughout the whole curve.*

This result may be written

$$\mu x \sin\phi = \mu_0 a \sin z \ldots\ldots\ldots\ldots\ldots\ldots (A),$$

μ_0, a, and z being the values of μ, x, and ϕ at the surface of the earth.

Again, $\phi - \phi'$, the elementary deviation of the ray at Q, may be represented by δr, if r be the whole refraction. Let $\mu' - \mu = \delta\mu$. Then

$$\mu \sin\phi = (\mu + \delta\mu) \sin(\phi - \delta r)$$

$$= (\mu + \delta\mu)(\sin\phi - \delta r \cos\phi),$$

$$0 = \delta\mu \sin\phi - \mu\delta r \cos\phi,$$

$$\frac{dr}{d\mu} = \frac{\tan\phi}{\mu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B).$$

Eliminating ϕ between (A) and (B), we get

$$\frac{dr}{d\mu} = \frac{1}{\mu} \frac{\mu_0 a \sin z}{\sqrt{(\mu^2 x^2 - \mu_0^2 a^2 \sin^2 z)}}$$

the *general differential equation of refraction.* This equation, however, cannot be integrated, as we do not know the connection between x and μ.

237. All that can now be done, in order to complete the solution, is to assume hypothetically a relation between μ and x, then integrate the equation, and, having made a sufficient number of direct observations of refraction to determine the

constants involved, consider that hypothesis as most nearly representing the true state of the case which gives results most in accordance with all other observations.

Bessell's hypothesis satisfies this condition, but the investigation is too intricate for an elementary work.* We shall consider Simpson's hypothesis, which is much simpler, and from it we shall deduce Bradley's formula, which gives very correct values so long as the zenith distance does not exceed 85°.

Simpson's Formula.

238. Simpson assumed that some power of the coefficient of refraction varies inversely as the distance from the centre of the earth, or

$$\mu^{n+1} \propto \frac{1}{x},$$

$$\mu^{n+1} x = C,$$

where n is a constant to be determined; therefore

$$\frac{n+1}{\mu} + \frac{1}{x}\frac{dx}{d\mu} = 0,$$

and from (A) $$\frac{1}{\mu} + \frac{1}{x}\frac{dx}{d\mu} + \frac{1}{\tan\phi}\frac{d\phi}{d\mu} = 0;$$

therefore $$\frac{n}{\mu} = \frac{1}{\tan\phi}\frac{d\phi}{d\mu},$$

whence, by (B), $\dfrac{dr}{d\phi} = \dfrac{1}{n}.$

To determine the limits of integration, we remark that the earth's atmosphere, at a few miles distance from the surface, becomes so rarified that its action on the path of the ray may be neglected, and we may consider it as bounded by a limiting sphere where the density is so small that the coefficient of refraction is 1.

* We may refer the student to Chauvenet's *Astronomy*, vol. I., and to Brunnow's *Spherical Astronomy*.

Let ψ be the value of ϕ at the limiting sphere, and x' the value of x; then

$$x' \sin\psi = \mu_0 a \sin z, \text{ and } x' = a\mu_0^{n+1},$$

whence
$$\mu_0^n \sin\psi = \sin z,$$

and
$$r = \int_\psi^z \frac{d\phi}{n} = \frac{z-\psi}{n},$$

$$r = \frac{z - \sin^{-1}\left(\dfrac{\sin z}{\mu_0^n}\right)}{n},$$

this is Simpson's formula of refraction.

Bradley's Formula.

239. From Simpson's formula we get

$$\frac{\sin z}{\sin(z-nr)} = \mu_0^n;$$

therefore
$$\frac{\sin z - \sin(z-nr)}{\sin z + \sin(z-nr)} = \frac{\mu_0^n - 1}{\mu_0^n + 1},$$

$$\tan\frac{nr}{2} = \frac{\mu_0^n - 1}{\mu_0^n + 1} \tan\left(z - \frac{nr}{2}\right),$$

or, approximately,
$$r = \frac{2}{n}\frac{\mu_0^n - 1}{\mu_0^n + 1} \tan\left(z - \frac{nr}{2}\right),$$

$$r = \alpha \tan\left(z - \frac{nr}{2}\right),$$

which is Bradley's formula.

The numerical values given by him were

$$r = 57''\cdot036 \tan(z - 3r),$$

corresponding to a mean state of the atmosphere, when the barometer stands at 29·6 inches, and the thermometer at 50° Fahr.

For zenith distances not exceeding 45°, we have very approximately

$$\text{ref.} \propto \tan(\text{zenith dist.}).$$

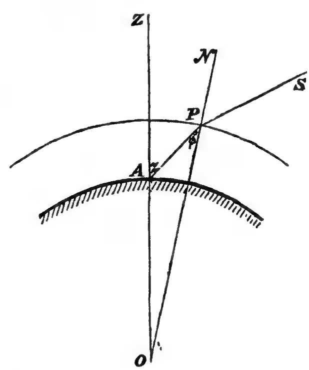
Cassini's Formula.

240. A formula of refraction based on the supposition of a homogeneous atmosphere was obtained by Dominique Cassini. Although not representing the actual state of nature, it gives tolerably satisfactory results for zenith distances not exceeding 80°, and is interesting as being the first attempt to determine refractions theoretically.

Cassini assumed that if the atmosphere were replaced by one of the same uniform density as at the surface of the earth, and of such a definite height as to produce the same pressure at the surface, the refractions would be approximately the same.

On this supposition the ray is bent only at its entrance into the spherical shell, and the formula is easily obtained.

Let SPA be the ray bent at P,
$OA = a$ the earth's radius,
$OP = a(1+n)$ where n is very small,
$\phi' + r = SPN$ the angle of incidence,
$\phi' = OPA$,
$z = PAZ$ the apparent zenith distance.



By the laws of refraction

$$\sin(\phi' + r) = \mu \sin\phi',$$

or, r being small,

$$\sin\phi' + r\cos\phi' = \mu\sin\phi';$$

therefore $$r = (\mu - 1)\tan\phi';$$

and, from the triangle OAP,

$$\sin\phi' = \frac{\sin z}{1+n};$$

therefore $$r = (\mu - 1)\frac{\sin z}{\sqrt{\{(1+n)^2 - \sin^2 z\}}},$$

$$r = \frac{(\mu - 1)\sin z}{\sqrt{(\cos^2 z + 2n)}} \text{ neglecting } n^2,$$

$$r = \frac{(\mu - 1)\tan z}{\sqrt{(1 + 2n\sec^2 z)}},$$

$$r = (\mu - 1)\tan z\,(1 - n\sec^2 z).$$

241. The preceding results have all been obtained without taking into account the changes in the state of the atmosphere, and the refraction has been considered as dependent only on the apparent zenith distance; whereas the density of the air, which is continually changing, must have considerable influence.

The values given by the formulæ must be considered as applying to some mean state of the atmosphere, and we must make corrections for alterations in the height of the thermometer and barometer, by assuming that the refraction varies as the density.

Let r be the tabulated refraction corresponding to a zenith distance z, the barometer standing at some definite height h, the temperature of the mercury as shewn by the attached thermometer being t, and the temperature of the air given by an external thermometer T.

Suppose that the refraction r' is required when the same zenith distance z is observed, the values of the quantities being respectively h', t', and T'.

Let d and p be the density and pressure of the air for the tabulated values h, t, T,

d' and p' for the observed values h', t', T'.

Then $r' : r :: d' : d$,

$$\left.\begin{aligned} d' : d &:: \frac{p'}{1+ET'} : \frac{p}{1+ET}, \\ p' : p &:: \frac{h'}{1+et'} : \frac{h}{1+et}; \end{aligned}\right\} \quad \text{(Besant's } \textit{Elementary Hydrostatics}\text{)}$$

therefore $r' : r :: \dfrac{h'}{(1+ET')(1+et')} : \dfrac{h}{(1+ET)(1+et)}$,

and $$r' = r\frac{h'}{h}\{1 - E(T' - T) - e(t' - t)\},$$

where E is the coeff. of expansion of air for 1° Fahr. $= \cdot 002036$,
e .... mercury .... $= \cdot 0001001$.

Coefficient of Refraction Determined by Observation of a Circumpolar Star.

242. We shall consider that the refraction r is of the form au, u being a known function of the observed zenith distance, and of the barometer and thermometer readings, and a the constant coefficient of refraction.

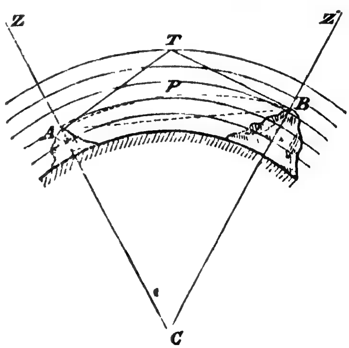
Observe a circumpolar star at its upper and lower transits; and let z, z' be the two zenith distances obtained, c the co-latitude of the place.

Then $$c = \tfrac{1}{2}\{(z + r) + (z' + r')\}$$
$$= \tfrac{1}{2}\{(z + z') + a(u + u')\}.$$

A second circumpolar star will, in the same manner, give
$$c = \tfrac{1}{2}\{(z_1 + z_1') + a(u_1 + u_1')\};$$
therefore $$z + x' + a(u + u') = z_1 + z_1' + a(u_1 + u_1'),$$
$$a = \frac{(z_1 + z_1') - (z + z')}{(u + u') - (u_1 + u_1')}.$$

Terrestrial Refraction.

243. Let A, B be two places on the surface of the earth, each visible from the other, and let APB be the curved path of the ray which connects them, AB the chord.



Then if AT, BT be the tangents to the path at A and B, and AZ, BZ' be verticals, the apparent zenith distances of the two places as seen one from the other will be

$ZAT = z$, and $Z'BT = z'$; and the refractions, due to the atmosphere, will be $TAB = r$, and $TBA = r'$.

The arc APB being a small portion of a curve of finite curvature, the angles r and r' are approximately equal; therefore

$$2r + z + z' = 180 + C,$$

$$r = \frac{180 + C - z - z'}{2} \quad \text{.................. (i)},$$

which determines r; z and z' being known by observation, and C by measurement of the distance AB and the known radius of the earth.

According to Biot,

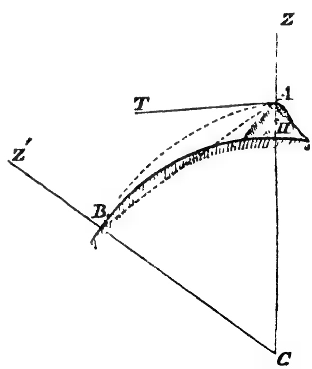
$$r = \alpha C,$$

where α depends on the state of the atmosphere; therefore

$$\alpha = \frac{180 + C - z - z'}{2C} \quad \text{..................(ii)}.$$

A large number of experiments has given 0·078 for the mean value of the coefficient α. The extreme values being 0·05 in summer and 0·15 in winter.

244. In Art. 8 (note), it was stated that refraction diminished the dip, and increased the distance, of the visible horizon.



Let A be the observer at a height $AH = h$ above the sea level, AB the curved ray by which the horizon is seen at B, C the corresponding angle at the earth's centre, $r = \alpha C$ the refraction at B and A.

Draw the straight line BA.

Then $\qquad Z'BA = 90° + r;$

therefore $\qquad ZAB = 90° - r + C,$

and, calling a the radius of the earth,

$$\frac{a+h}{a} = \frac{\sin(90+r)}{\sin(90-r+C)} = \frac{\cos r}{\cos(C-r)},$$

$$\frac{h}{a} = \frac{\cos r - \cos(C-r)}{\cos(C-r)} = \frac{2\sin\frac{1}{2}C\sin(\frac{1}{2}C-r)}{\cos(C-r)},$$

and, C and r being small angles,

$$\frac{2h}{a} = \frac{C(C-2r)}{1-\frac{1}{2}(C-r)^2} = C^2\left(1-\frac{2r}{C}\right),$$

$$\sqrt{\left(\frac{2h}{a}\right)} = C\left(1-\frac{r}{C}\right) = C(1-\alpha).$$

If C_1 be the value of C, supposing no refraction,

$$C_1 = \sqrt{\left(\frac{2h}{a}\right)};$$

therefore $C = C_1(1+\alpha) = C_1(1{\cdot}078) = C_1 + \frac{1}{13}C_1$ approximately.

245. Let D, D_1 be the corresponding values of the depression: we find, AT being the tangent to the visual ray at A,

$$D = ZAT - 90° = ZAB - r - 90° = C - 2r$$
$$= C(1-2\alpha),$$

but $D_1 = C_1 = C(1-\alpha)$;

therefore $D = D_1\dfrac{1-2\alpha}{1-\alpha} = D_1(1-\alpha) = D_1 - \frac{1}{13}D_1$.

There are, however, considerable irregularities in these values from day to day; and errors in the altitude of the sun or of a star, at sea, may frequently be traced to an unknown, and therefore uncorrected, change in the dip of the sea-horizon from which the altitudes are measured.

Other Effects of Refraction.

246. If we examine a table of refractions, we shall find that with the barometer at 30 inches, and Fahrenheit's thermometer at 50°—

At apparent zen. dist. 45°	the refraction is	0′ 58″·2,
.... 80°	....	5′ 19″·2,
.... 85°	....	9′ 52″,
.... 88°	....	18′ 26″,
.... 90°	....	36′ 29″.

The change is very rapid near the horizon, and a consequent contraction of the vertical diameters of the sun and moon takes place, giving them a sensibly oval shape just after rising or before setting, the lower half being somewhat more flattened than the upper half. For example: suppose the true altitude of the sun's lower limb to be 5° and his diameter 32′, we find

	lower limb.		upper limb.
True altitude	5° 0′ 0″	......	5° 32′ 0″
Refraction	9′ 52″	......	8′ 52″
Apparent alt.	5° 9′ 52″	......	5° 42′ 52″

the difference of which gives an apparent vertical diameter of 31′, or a contraction of 1′. When nearer the horizon the contraction may extend to 5′ or 6′.

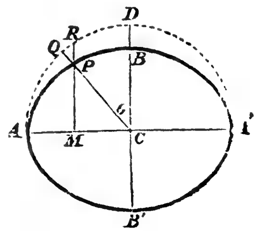
247. *To find the contraction, produced by refraction, of a semi-diameter which makes an apparent angle θ with the vertical.*

Let ACA' be the horizontal diameter of the oval disk,

BCB' the vertical one,

CP the semi-diameter making an angle θ with CB,

PM the ordinate of P.



The contractions of the ordinates are approximately proportional to their magnitudes.

Produce CB, CP, MP to meet the auxiliary circle in D, Q, and R,

$$PR : BD :: MP : CB,$$

$$:: MP : CP \text{ approximately};$$

therefore $$PR = BD \cos\theta.$$

Again, the small triangle PQR right-angled at Q gives

$$PQ = PR \cos\theta,$$

therefore $$PQ = BD \cos^2\theta,$$

or, the contraction of any semi-diameter is equal to that of the vertical semi-diameter multiplied by the square of the cosine of the included angle.

248. The horizontal diameter itself will be slightly diminished by refraction. This is obvious, without special investigation, by merely remarking that the extremities of the horizontal diameter are equally raised by refraction, each in its own vertical, and these verticals meet in the zenith—therefore the breadth must contract. This contraction is nearly constant for all altitudes of the sun or moon, and is about $0''\cdot5$.

Effect on the Rising and Setting of Heavenly Bodies.

249. Another effect of refraction is to accelerate the rising, and to retard the setting, of all bodies. The exact amount of this effect may be calculated by solving the usual spherical triangle SPZ, where $SZ = 90° + r$, and comparing the hour angle with that obtained when $SZ = 90°$.

In the same way, the azimuth at apparent rising and setting will place the body more to the north (in our hemisphere) than if there were no atmosphere; but the great irregularities in the values of r, for these low altitudes, render the investigation of little practical value.

CHAPTER XVI.

FIGURE OF THE EARTH. PARALLAX.

250. THE determination of the exact figure of the earth is a problem of considerable difficulty, but of great importance. As stated in Art. 27, if a small arc of a meridian be measured on the surface, and also the difference of latitude of the two extremities of the arc, or, which is the same thing, the change in the meridian zenith distance of a star, we shall thus have an arc and the angle it subtends at the centre of the earth, whence the radius may be found.

This, however, supposes the earth to be spherical; and a more correct statement would be, that the value so obtained is the radius of curvature of the meridian at the middle point of the small arc. Now, by actual measurement of such arcs in various latitudes, it has been found that the radius of curvature increases, or, in other words, that the curvature diminishes, and the earth becomes more flattened, as we approach the poles.

The figure of the earth is found to be very approximately an oblate spheroid formed by the revolution, about its minor axis, of an ellipse whose semi-axes are respectively

$$a = 3962{\cdot}8 \text{ miles},$$

$$b = 3949{\cdot}6 \text{ miles},$$

the axis of figure coinciding with the polar axis of the earth.

These values of a and b are mean values; for, different meridians present slight discordances,—so slight, however, that we may here neglect them. For an account of the

niceties and precautions required in the measurement of the arcs and angles, and for an explanation of other methods which have been employed in the determination of the figure of the earth, we must refer to works on geodesy.

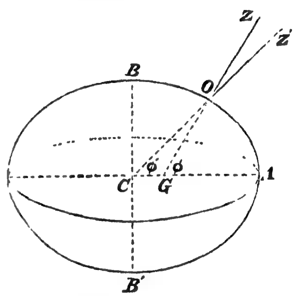
251. The fraction $\frac{a-b}{a}$ is called the *compression*. If we represent it by c, and the excentricity by e, we shall have

$$\frac{b}{a} = 1 - c,$$

$$e^2 = 1 - \frac{b^2}{a^2} = 2c - c^2.$$

The value of c is $\frac{1}{300}$ nearly, and $e = \cdot 0816$.

252. Let BOA be the meridian of a place O on the surface of the earth,



BCB' the polar axis, CA the equatoreal radius, and OG the normal at O. Then (Art. 29),

OGA is the geographical latitude, or ϕ,

OCA is the geocentric latitude, or ϕ',

and COG is the reduction of the latitude $= \phi - \phi'$.

If x, y be the coordinates of O,

$$\tan\phi' = \frac{y}{x}, \quad \tan\phi = \frac{y}{x(1-e^2)};$$

therefore $$\tan\phi' = (1-e^2)\tan\phi = \frac{b^2}{a^2}\tan\phi \text{..........(i)}$$

is the connection between the geocentric and the geographical latitudes;

$$\tan(\phi - \phi') = \frac{\tan\phi - (1-e^2)\tan\phi}{1 + (1-e^2)\tan^2\phi} = \frac{e^2 \sin\phi \cos\phi}{1 - e^2\sin^2\phi},$$

or, approximately, $$\phi - \phi' = c \sin 2\phi \text{....................(ii)},$$

which gives the *reduction*.

253. We shall also determine the distance CO. From above

$$\frac{y}{x} = \frac{b^2}{a^2} \tan\phi,$$

$$\frac{\frac{y}{b}}{b \sin\phi} = \frac{\frac{x}{a}}{a \cos\phi},$$

$$\frac{\left(\frac{y}{b}\right)^2}{b^2 \sin^2\phi} = \frac{\left(\frac{x}{a}\right)^2}{a^2 \cos^2\phi} = \frac{1}{a^2 \cos^2\phi + b^2 \sin^2\phi},$$

therefore

$$y^2 = \frac{b^4 \sin^2\phi}{a^2 \cos^2\phi + b^2 \sin^2\phi}, \quad x^2 = \frac{a^4 \cos^2\phi}{a^2 \cos^2\phi + b^2 \sin^2\phi},$$

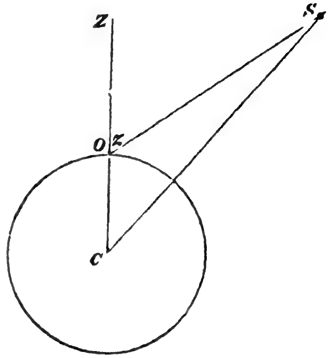
therefore $$CO^2 = \frac{a^4 \cos^2\phi + b^4 \sin^2\phi}{a^2 \cos^2\phi + b^2 \sin^2\phi},$$

whence $$CO = a\,(1 - e\,\sin^2\phi), \text{ approximately.}$$

Parallax.

254. There are some bodies so remote from us that, whether seen from the centre or from any point on the surface of the earth, the two directions will be so nearly parallel that no instrument we at present possess can measure, or even detect, their inclination. There are others within, what may be termed, a measurable distance, in which the difference of direction is, although small, a quantity which may be determined. It is therefore essential, in order to make the registered right ascensions, declinations, &c., available to all persons, that they should be referred to some definite point; and it will obviously be advantageous to select the centre of the earth for point of reference, because the apparent motions of those bodies, as seen from the centre, are of a much more simple character than as seen from any point of the surface, and also because we can much more readily reduce the observations from any point to the centre than to another point. The declinations, &c., of all bodies re-

gistered in the Nautical Almanac, are those they would have as seen from the centre of the earth; and when we make an observation at any place we must, when there is need, know how to transform it to what it would be at the centre. This is called the correction for parallax.



The parallax of a heavenly body is the angle between the directions of two lines drawn to it, one from the observer, the other from the centre of the earth; or, in other words, the angle subtended at the celestial body by that radius of the earth which is drawn to the observer.

Thus, if C and O be respectively the centre and the observer, S a distant object, the angle CSO is the parallax of S.

255. We shall firstly consider the earth as a sphere; then CO produced will pass through the zenith of O, and the effect of parallax will be wholly in the vertical plane ZOS; it will change the zenith distance from ZCS to ZOS, the difference between them being the parallax itself CSO.

If a be the radius of the earth, D the distance CS,
z the zenith distance ZOS,
p the parallax CSO,

$$\sin p : \sin z :: a : D,$$

$$\sin p = \frac{a}{D} \sin z.$$

When $z = 90°$, p becomes the horizontal parallax: let us represent it by Π, then

$$\sin \Pi = \frac{a}{D};$$

therefore $$\sin p = \sin \Pi \sin z.$$

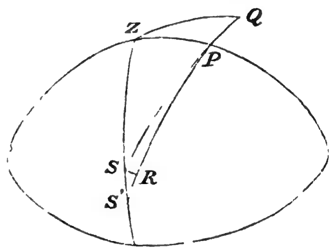
Except in the case of the moon, whose parallax sometimes exceeds 1°, we may substitute the angles for their sines and write

$$p = \Pi \sin z.$$

256. The effect of parallax on any coordinate of the body's position is generally spoken of as parallax in that coordinate, thus:—parallax in declination, parallax in hour angle, &c. The foregoing article shews that parallax in altitude or in zenith distance is the whole parallax itself, and that parallax in azimuth is zero.

To find the Parallax in Declination and Hour-angle.

Let P, Z, S' be the pole, the zenith, and the body, on the celestial sphere of the observer at the surface of the earth. Suppose the observer transferred to the centre, and let S be the new position of the body in the vertical ZS', then



$$SS' = \Pi \sin ZS';$$

and if h and δ be the hour-angle and declination, referred to the centre; $h + \alpha$, $\delta - \beta$ the same, referred to the surface; draw the arc SR perpendicular to PS', then

$$SPS' = \alpha, \quad S'R = \beta.$$

$$\alpha = \frac{SR}{\sin PS} = \frac{SS' \sin SS'R}{\sin PS} = \frac{\Pi \sin ZS' \sin SS'R}{\sin PS},$$

$$\alpha = \frac{\Pi \sin ZP \sin ZPS'}{\sin PS}$$

$$= \frac{\Pi \cos\phi \sin(h + \alpha)}{\cos\delta},$$

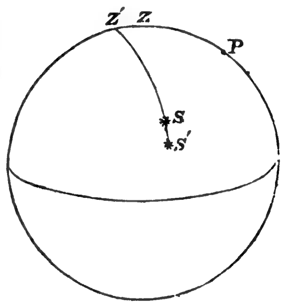
ϕ being the latitude of the observer.

Again, in $S'P$, produced if necessary, take $S'Q=90°$, therefore $PQ=\delta-\beta$, then

$$\beta = S'R = SS' \cos SS'R = \Pi \sin ZS' \cos ZS'Q$$
$$= \Pi \cos ZQ$$
$$= \Pi (\cos ZP \cos PQ + \sin ZP \sin PQ \cos ZPQ)$$
$$= \Pi \{\sin\phi \cos(\delta-\beta) - \cos\phi \sin(\delta-\beta) \cos(h+\alpha)\}.$$

257. When the spheroidal form of the earth is taken into account, the problem is only slightly altered.

When the observer is transferred to the centre the apparent displacement $S'S$ of a body S' no longer takes place towards the geographical zenith Z, but towards the geocentric zenith Z' where $ZZ'=\phi-\phi'$ the difference between the geocentric and geographical latitudes (Art. 252).



The value of SS' will be given by

$$\sin S'S = \frac{CO}{D} \sin Z'S', \text{ as in Art. 255},$$

$$= \frac{a}{D}(1 - c\sin^2\phi) \sin Z'S' \text{ (Art. 253)},$$

since a is the equatorial radius, $\frac{a}{D}$ is the sine of the *equatorial horizontal parallax*, which is the element registered in the Nautical Almanac, and the parallax Π in the geocentric horizon of the place will be given by

$$\sin \Pi = \frac{a}{D}(1 - c\sin^2\phi).$$

The figure shews that, except when the body is in the meridian, there will be parallax in azimuth as well as in altitude; and a comparison with the figure in the previous article also shews that these parallaxes may be determined

by means of the formulæ which give α and β in that article, if we use $90° - ZZ'$ for ϕ and the observed azimuth for $h + \alpha$.

And again, the parallaxes in hour-angle and in declination will be given by the same formulæ, if we use the geocentric instead of the geographical latitude.

258. The formula $\sin(\textit{equat. horiz. par.}) = \frac{a}{D}$, in which a is the known equatorial radius of the earth, shews that the determination of the parallax of a body is the same problem as the determination of its distance.

For all bodies, except the moon, the distance D is very great as compared with a, and the parallax consequently very small; so that all the refinements of which modern astronomy is capable are necessary to determine it by observation. So much is this the case, that the parallax of the sun, which was supposed to have been determined with great accuracy by the transit of Venus in 1769, has within the last few years been found to be in error probably by about $\frac{1}{25}$th of itself.

259. *To determine the parallax of a heavenly body by meridian observations.*

Let A, B be two stations in opposite hemispheres, on the *same* meridian but in widely different latitudes, and S the body whose parallax is required.

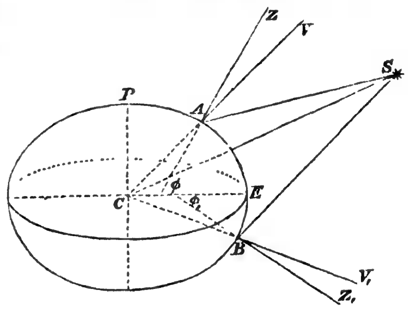
We must firstly remark that in the figure, for the sake of distinctness, the earth is immensely exaggerated in comparison with the distance of S; so that the lines AS, BS are nearly parallel, and the apparently large angle ASB is in reality a very small angle not exceeding $1'$, except in the case of the moon where it may reach from $1°$ to $2°$.

The small angle ASB is obviously the change in the declination of S, as seen from A and from B. We have to find its value with the greatest accuracy, but we cannot do this by a comparison of the absolute declinations deduced

from observed zenith distances, because the observed directions of S will be affected by refraction, and the correction from the refraction tables will always be somewhat uncertain. To avoid this source of error and also any errors of the divided circle, a star is selected whose declination is so nearly the same as that of S, that both may pass through the field of view of the mural or transit circle in a fixed position of the instrument, and the difference of declination may then be measured by means of the micrometer.

The very small difference of refraction due to the slight difference of zenith distance of the two bodies may be accurately found and allowed for; and thus, both S and the star will be affected with the same errors with the exception of parallax which does not affect the star, and the angle obtained will be the true difference of their declinations as seen from that station.

If at the other station similar observations be made, and the position of S be compared *with the same star* on the same day and therefore at the same time, the difference of their declinations as seen from that station will also be found, and the change in that difference will be the angle ASB.



Let m be the value of ASB thus obtained

ϕ, ϕ_1 the geographical latitudes of A and B.

ϕ', ϕ_1' their geocentric latitudes;

r, r_1 the radii CA, CB;

a the equatorial radius, c the compression;

D the distance CS;

Π_0 the equatorial horizontal parallax;

z, z_1 the observed zen. dist. ZAS, Z_1BS.

Produce CA, CB to V, V_1, then

$$\sin ASC = \frac{r}{D} \sin VAS = \frac{a}{D}(1 - c \sin^2\phi) \sin VAS$$

$$= \sin \Pi_0 (1 - c \sin^2\phi) \sin (z - \phi + \phi') \ldots\ldots\ldots\ldots\ldots \text{(i)},$$

$$\sin BSC = \sin \Pi_0 (1 - c \sin^2\phi_1) \sin (z_1 - \phi_1 + \phi_1') \ldots\ldots\ldots\ldots \text{(ii)},$$

and $$ASC + BSC = m \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}.$$

Eliminating ASC and BSC between these three equations, the value of Π_0 will be obtained in terms of known quantities.

Except in the case of the moon, we may replace the sines of the small angles by the angles themselves,

$$\Pi_0 = \frac{m}{(1 - c \sin^2\phi) \sin (z - \phi + \phi') + (1 - c \sin^2\phi_1) \sin (z_1 - \phi_1 + \phi_1')}.$$

260. We have taken the two places A and B on the same meridian, but this condition could with difficulty be secured, and is moreover not essential. For, by taking account of the change of declination of S during the interval between its transit over the two meridians, we can, by simply adding or subtracting this change, reduce the difference of declination between S and the star, observed at either place, to what it would have been had the meridians coincided.*

261. By this method the parallax of the moon has been determined, and recently that of the planet Mars when in opposition. The sun and the other planets are too far away to allow us to apply the method directly to them, but by Kepler's third law (Art. 189), if the distance of one planet from the earth be known, that of the sun and of the other planets may be inferred.

The planet Mars was in a favourable position for the determination of its parallax in 1862, and the result of the

* For further development of this and other methods, we shall refer to Chauvenet's *Astronomy*, vol. I. The Rev. R. Main's *Practical and Spherical Astronomy*, and Woodhouse's *Astronomy*.

observations then made has necessitated a change in the hitherto received value of the sun's parallax, altering it from 8″·57 to 8″·93. This must, however, be looked upon as only provisional, until the transit of Venus in December next, a phenomenon far better adapted than any other to the determination. The value 8″·57 was found by the transit of 1769, and any error must be attributed to the imperfection of the instruments, &c., then in use; for, the method admits of such precision, that even now the recent determination by observations of Mars would not have led to the rejection of the former value, if the new one had not been confirmed in a remarkable manner by independent observations made on the velocity of light. We shall, in a future chapter, give a brief sketch of the method of determining the sun's parallax by the transit of Venus.*

Distances of the Sun and Moon.

262. The parallax being known, we may deduce the distance by the formula (Art. 258),

$$\text{distance} = \frac{\text{radius of earth}}{\sin(\text{hor}^{\text{l}}.\ \text{par}^{\text{x}}.)}.$$

$$\text{Sun's distance} = \frac{\text{radius of earth}}{\sin 8''\cdot 93} = \frac{206265}{8\ 93}\ (\text{radius}),$$

$$= 23098\ (\text{earth's radius}),$$

$$= 91533000\ \text{miles}.$$

In the case of the moon whose mean parallal is 57′ 1″·8,

$$\text{mean distance} = 60\ (\text{earth's equatorial radius}),$$

$$= 237800\ \text{miles}.$$

The moon's distance from the earth is therefore only $\frac{1}{400}$ of the sun's distance.

* See an article on "Celestial Measurings," by Sir John Herschel, in *Good Words* for June, 1864.

Magnitudes of the Sun and Moon.

263. If we measure the angular semi-diameter of a heavenly body whose parallax is known, we can determine its magnitude

$$\sin(\text{semi-diam.}) = \frac{\text{radius of body}}{\text{distance}},$$

$$\sin(\text{parallax}) = \frac{\text{radius of earth}}{\text{distance}};$$

therefore $$\frac{\text{radius of body}}{\text{radius of earth}} = \frac{\sin(\text{semi-diam.})}{\sin(\text{parallax})}.$$

In the case of the sun, semi-diameter $= 16'$, parallax $= 8''\cdot 93$,

radius of sun $= 108$ (earth's radius) $= 428000$ miles.

In the case of the moon, semi-diam. $= 15' 39''\cdot 9$ parallax $57' 1''\cdot 8$,

radius of moon $= \frac{3}{11}$ (earth's radius) $= 1080$ miles.

Since the moon's distance from the earth is only sixty times the earth's radius, we see that the magnitude of the sun is such that, if it were concentric with the earth, it would include the moon's orbit and extend nearly as far again beyond.*

Distances of the Stars. Annual Parallax.

264. The stars are too far off to allow of any measurement of their distances being made by the foregoing means. But we have now got the distance of the sun, and although a

* The mass of the sun is determined as follows: If a be the radius of the earth, E its mass, and g the acceleration of gravity at the surface; also, if D be the distance of the sun, S its mass, and if one year be T, then

$$g = \frac{E}{a^2}, \quad T = \frac{2\pi D^{\frac{3}{2}}}{S^{\frac{1}{2}}} \text{ (Newton, Sect. III.)},$$

therefore $$\frac{S}{E} = \frac{4\pi^2 D^3}{g T^2 a^2},$$

whence S may be expressed in terms of E, and is found about $322000E$.

The mass of the moon is obtained by its effect on the earth's nutation, and is estimated at about $\frac{1}{80}E$.

globe of 8000 miles diameter subtends no appreciable angle at the nearest star, we may expect that a displacement of 183,000,000 miles—which, being the breadth of the earth's orbit, is the distance that separates two positions of the observer at six months' interval—will be sufficient to cause a measurable change in the apparent direction of some of the stars.

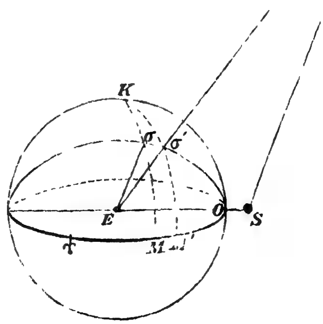
The maximum angle which the radius of the earth's orbit subtends at any star is called the *annual parallax* of that star.

265. *The annual parallax of a star being given, to determine the parallax in longitude and latitude, at a given time*, that is, the difference between these coordinates, as observed from the earth, and as they would be, if observed from the sun.

Let E be the earth, S the sun, s a star whose annual parallax p is known.

About E describe the observer's celestial sphere (Art. 11). Let ♈MO be the ecliptic, K its pole, O and σ' the places of the sun and star.

Draw $E\sigma$ parallel to Ss, and therefore meeting the sphere in the great circle through O and σ'.



Draw also the quadrants $K\sigma M$, $K\sigma' M'$. Then $\sigma\sigma'$ is the displacement of the star owing to parallax, MM' will be its effect on the longitude, and $M\sigma \sim M'\sigma'$, on the latitude of the star.

Let $$SE = r, \text{ and } Ss = \Delta,$$

$$\sin s = \frac{r}{\Delta} \sin SEs,$$

or $$\sigma\sigma' = p \sin \sigma' O,$$

where p, or $\frac{r}{\Delta}$, is the annual parallax, *i.e.* the maximum value of $\sigma\sigma'$, corresponding to that position of E where SEs is a right angle. From this we obtain

$$MM' = \frac{\sigma\sigma' \sin \sigma}{\cos M'\sigma'} = \frac{p \sin \sigma' O \sin \sigma'}{\cos M'\sigma'}, \text{ since } \sigma' = \sigma, \text{ very nearly,}$$

$$= \frac{p \sin M'O}{\cos M'\sigma'} = \frac{p \sin (\Upsilon O - \Upsilon M')}{\cos M'\sigma'};$$

therefore parallax in longitude $= \dfrac{p \sin(\odot - *)}{\cos(\text{lat. of star})}$ ....(i),

where $\odot$ is the longitude of the sun,

and $*$ .... star.

Again, parallax in latitude

$$= -\sigma\sigma' \cos \sigma'$$

$$= -p \sin \sigma' O \cos \sigma'$$

$$= -p \sin \sigma' M' \cos M'O$$

$$= -p \sin(\text{lat. of star}) \cos(\odot - *) \text{ .... (ii).}$$

Equations (i) and (ii) shew that, owing to annual parallax, the displacement of a star from a mean position will extend from $-p$ to $+p$ in a direction parallel to the ecliptic, and from $-p$ sin(lat.) to $+p$ sin(lat.) in a perpendicular direction; the apparent path of the star, during the sidereal year, being a small ellipse which has the above displacements for major and minor axes respectively.

266. Such is the immense distance of the stars that, out of a large number that have been attentively examined, only a very few shew any sensible parallax. For the nearest of these, the annual parallax p is less than $1''$; and

the determination of such a minute quantity requires the best instruments and the greatest precision. On account of the interest of the subject, we shall here briefly describe the two methods by which this measurement has been effected.

To determine the annual parallax of a fixed star by observation.

First Method. Let the star's longitude be observed when that of the sun differs about 90° from it; and let the observation be repeated six months later, when, the earth having moved to the opposite side of its orbit, the sun's longitude again differs by 90° from that of the star.

Equation (i) shews that the parallax in longitude at the two observations will be respectively

$$p \sec(\text{star's lat.}) \text{ and } -p \sec(\text{star's lat.}).$$

Therefore, if the two observed longitudes of the star differ by 2δ, we shall have

$$2\delta = 2p \sec(\text{star's lat.});$$

therefore $$p = \delta \cos(\text{star's lat.}).$$

As the accuracy of δ depends on the accuracy with which the longitude is determined at each observation, and this again depends on the perfection of the instruments, on their stability, and on the care with which all corrections are made and all sources of error guarded against or taken into account, it can obviously only be after multiplied observations of the most delicate kind that a star can be asserted to have, or not to have, a measurable parallax.

The star α *Centauri*, a bright star in the southern hemisphere, was, by direct observations made at the Cape of Good Hope during the years 1832 (by Mr. Henderson) and 1839 (by Sir T. Maclear), found to have a parallax which was estimated at $0''\cdot98$.

267. A *Second Method*, due to Bessel, is free from many of the difficulties inherent to the first:—If two neighbouring stars differ very much in brightness, we may presume that they are at very different distances from us; and if we assume the smaller star to be so remote as to have no appreciable parallax, we may attribute any observed changes in the angular distance between them to the parallax of the nearer star—provided such changes go through a yearly cycle according to the parallactic law—and, from these changes, we may obtain the parallax itself.

The comparison star must be so situated that both may be seen at the same time in the field of view, and the angular distance between them may then be measured with very great accuracy by using the double image micrometer (Art. 112).

The great advantage of this method over the former is the fact of its eliminating all uncertainty of refraction, all errors from a want of stability of the instrument, and its avoiding a number of corrections which have to be made for the determination of the absolute longitude.*

Bessel applied this method with success to the star 61 *Cygni*, not a very bright star, being only of the fifth magnitude, but having near it, at distances of about 8′ and 12′ respectively, two much smaller stars between the ninth and tenth magnitudes. These were well situated also as to direction, being from 61 *Cygni* nearly at right angles to one another. Bessel found nearly the same parallax from each of these, and gave as the result of his observations 0″·35. Messrs. Auwers and Struve, with more perfect instruments, have repeated the observations, and the mean of their results, which agree very nearly, gives 0″·54.

This is unquestionably the best determined star-parallax. It corresponds to a distance 382,000 times that of the sun.

* For a full investigation of the method we may refer to Chauvenet's *Astronomy*, vol. I. p. 693.

268. The following are some of the stars whose parallaxes have been determined by one or other of these two methods. These are our nearest neighbours, and to convey a better idea of the enormous distances which separate us from them, we have given the time that their light takes to travel to us; it being known that light travels at the rate of 184,000 miles in a second, and takes $8^m\ 18^s$ to come from the sun to the earth (see Chap. XVII.).

	Parallax.	Distance from earth in radii of earth's orbit.	Time the light takes to reach the earth.
α Centauri	0″·98	210,000	3·315 years.
61 Cygni	0″·54	382,000	6·029
α Lyræ	0″·26	793,000	12·518
Sirius	0″·15	1,375,000	21·706
Arcturus	0″·127	1,624,000	25·637
Polaris	0″·106	1,946,000	30·720

Secular Parallax.

269. Many of the stars are found to have a *proper motion* which must not be confounded with the displacements due to annual parallax, or to those other causes of disturbance which we have yet to speak of. The essential distinction is, that these disturbances are periodic and the proper motion is not. A star which has proper motion is carried by it among the other stars at a uniform rate in a definite direction.

Sir W. Herschel found that these motions could, in nearly every case, be explained by supposing the sun, with its accompanying system of planets, &c., to be sweeping through space towards some distant point.

It is obvious, that if such a motion of the solar system exist, the stars, in the region towards which it is moving, must seem to open out and separate more and more, while those in the opposite direction must close up.

The point determined by Herschel was near the star λ *Herculis*, but, by the investigations of subsequent observers, and by employing a larger number of stars, a more accurate, though not widely different, position of the point has been obtained.

"The motion of the solar system in space is directed to a point in the celestial sphere, situate on the right line which joins the two stars of the third magnitude π and μ *Herculis*, at a quarter of the distance between them measured from the former."*

270. Astronomers have carried their speculations on the character of the sun's motion still further, and, reasoning from analogy, have supposed that his path in space is not a straight line, directed towards the point above determined, but a curve to which this line is a tangent; and that, in countless ages, the solar system describes a gigantic orbit round some central position or body.

Mädler, discussing the proper motions of the stars, assigns the position of central sun to *Alcyone*, one of the group of the *Pleiades*. Many ages must elapse before the confirmation of this statement can be received with certainty.

Supposing such a centre—then, the *secular parallax* of a star belonging to a yet more distant system would be the angle which the radius of this immense orbit would subtend at the star.

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* Struve.

CHAPTER XVII.

ABERRATION.

271. The light of the celestial bodies is not brought to the observer by instantaneous transmission, but by successive propagation; and this motion, combined with that of the earth in its orbit, is the cause of an apparent displacement of the bodies, to which the name of *aberration* has been given.

The existence of aberration is, as we shall presently shew, a necessary consequence of the velocity of the earth being comparable with that of light; but the ratio of the former velocity to the latter is so small (about 1 : 10000), that the resulting aberration—whose magnitude depends on this ratio—had remained hidden and unsuspected until detected and explained by Bradley about the year 1729.

272. The gradual propagation of light had been discovered by Roemer in 1675. He remarked that the eclipses of Jupiter's satellites always preceded their predicted times when the earth and Jupiter were on the same side of the sun, and happened later when on opposite sides. This he satisfactorily accounted for, by supposing light to require time for its transmission:—the predicted times—being determined from a large number of observations—would correspond to the mean distance of the planet from the earth; and therefore, in the first case, if the earth and planet were at their nearest distance, the extinction of the light would be known to the observer earlier than if he occupied his mean or average distance, by so much time as light would

take to move through the radius of the earth's orbit; and, in the second case, when at their furthest distance apart, it would be just as much later.

This important fact has been confirmed of late years by direct observation. Mons. Fizeau, by a most ingenious apparatus, has succeeded in measuring the actual velocity of light (see Arago's *Astronomy*, vol. IV.); and Mons. Foucault, with improved instruments, has repeated the observations and obtained results in close agreement with those obtained indirectly by other means.

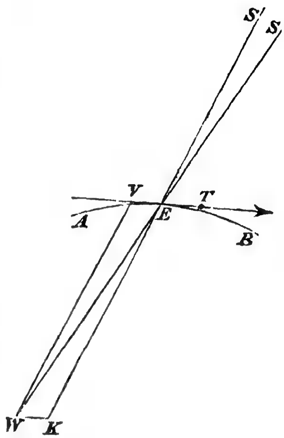
273. Bradley, about the year 1729, when observing certain stars, found apparent displacements which he could not account for by attributing them to any known cause. These displacements were periodical, and, as the periods were the same—one year—for all the stars observed, it was obvious that the orbital motion of the earth was in some manner concerned in producing them. After some failures, he hit upon the only hypothesis that seems able to account for the phenomenon.

The phenomenon itself is this: "All the stars seem to be displaced from their mean position towards that point of the heavens to which the direction of the earth's motion tends at the moment; and the amount of the displacement varies as the sine of the angle between the earth's direction and the line joining the earth and star—the constant multiplier, or *coefficient of aberration*, being the same for all stars."

274. To explain how this effect is produced by the combination of the motions of the earth and of light, suppose E to be the earth, AEB a part of its orbit, ES the true direction of a star.

Draw VET a tangent to the orbit at E. Take ET to represent the velocity of the earth in magnitude; and EK, in SE produced, to represent the velocity of light on the same scale.

Now, the relative motion will not be altered if any common velocity be given both to light and to the earth. Let a velocity EV, equal and opposite to ET, be so applied. Then the earth will be brought to rest at E, and light will have a velocity compounded of EV and EK, that is, a velocity EW, the diagonal of the parallelogram VK.



Hence the star instead of being seen in its true direction ES is seen in the direction ES', and the displacement SES' is the aberration.*

275. The aberration takes place in the plane SET, and towards the point to which the earth tends. The triangle WEK gives

$$\sin SES' : \sin S'ET = WK : EK,$$

or $$\sin(\text{aberration}) = \frac{\text{vel. of earth}}{\text{vel. of light}} \sin S'ET.$$

The aberration being small, we may write the circular measure for the sine, and also $\sin SET$ for $\sin S'ET$. The angle SET is called the *earth's way*, therefore

$$\text{aberration} = \frac{\text{vel. of earth}}{\text{vel. of light}} \sin(\text{earth's way}),$$

$$= k \sin(\text{earth's way}),$$

where k is the constant of aberration.

* The effect of aberration may also be *illustrated* in several ways: A man walking in a shower of rain when the rain drops fall vertically must hold his umbrella a little forward; the effect of his own motion being to make the rain beat in upon him, and this effect is the greater the faster he walks.

Again, suppose a shot from a battery to enter one side of a ship which is moving at right angles to the line joining the ship and the battery, and to go out at the opposite side. The two shot holes will not be immediately opposite one another,— the distance advanced by the ship during the passage of the shot through it will cause the exit to be further astern than the entrance; and a

276. The value of k is found as follows: By comparing numerous observations of the eclipses of Jupiter's satellites, it is inferred that light takes $8^m\ 18^s$ to travel from the sun to the earth; and during that time the earth will have described an arc of its orbit—supposed circular—whose length will be $\frac{8^m\ 18^s}{365\frac{1}{4}\text{ days}}\ 2\pi R$, where R is the radius of the orbit.

Therefore $$k = \frac{498 \times 2\pi}{365\frac{1}{4} \times 24 \times 60 \times 60},$$

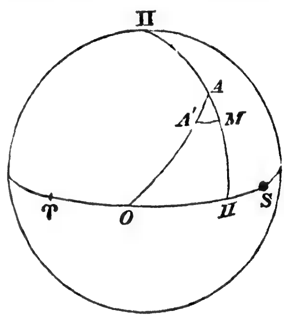
or, if expressed in seconds of arc,

$$k = \frac{498 \times 360 \times 60 \times 60}{365\frac{1}{4} \times 24 \times 60 \times 60} = \frac{498 \times 20}{487} = 20''\cdot 45.$$

This mean value of k is subject to small variations (not exceeding $0''\cdot 35$) due to the different velocities of the earth at different points of its orbit.

277. The effect of aberration will be to make the stars, when referred to the celestial sphere, describe small ellipses about their true places.

Let the accompanying figure represent the celestial sphere of the observer; ΥOS the ecliptic, Π its pole; A a star and S the sun; O a point on the ecliptic $90°$ behind S.



Since the direction of the earth's motion is at right angles to the line joining it with the sun (neglecting the excentricity of the orbit), it is obvious that O is the point of the ecliptic to which the earth tends, and therefore AO will be the earth's way for the star A. In AO take $AA' = k \sin AO$, A' will be the star's apparent place.

person on board, not allowing for the ship's motion, but judging of the position of the battery by the direction taken by the shot, would imagine it to be in advance of its true place.

If $A'M$ be drawn perpendicular to ΠAH, and if $A'M = x$ and $AM = y$,

$x = A'M = AA' \sin A'AM = k \sin AO \sin A'AM = k \sin OH$,

$y = AM = AA' \cos A'AM = k \sin AO \cos A'AM = k \sin AH \cos OH$,

therefore, $$\left(\frac{x}{k}\right)^2 + \left(\frac{y}{k \sin AH}\right)^2 = 1;$$

therefore the star describes an ellipse about its mean place, the semi-major axis being parallel to the ecliptic and equal to $20''\cdot45$, the semi-minor axis $20''\cdot45 \times \sin(\text{star's lat.})$.

278. This may also be shewn geometrically: Referring to the figure (p. 214), the line KW is parallel and equal to ET, which represents the earth's velocity; therefore, neglecting the small variations of velocity, W describes, round K as centre, a circle parallel to the plane of the ecliptic, and $S'EW$ describes an oblique cone on a circular base round the axis SEK.* The intersection of this cone by the celestial sphere of the observer will be approximately a plane curve, and therefore an ellipse.

279. *To determine the aberration of a star in latitude and longitude.*

Let ΥS the longitude of the sun $= \odot$ (fig. Art. 277),
ΥH the longitude of the star $= l$,
AH the latitude of the star $= \lambda$,

then, as above,

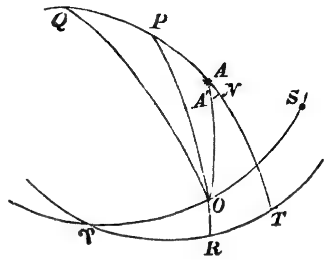
$$\begin{aligned}\text{aberration in latitude} &= -AM = -20''\cdot45 \sin AH \cos OH \\ &= -20''\cdot45 \sin\lambda \cos(l - \Upsilon O) \\ &= -20''\cdot45 \sin\lambda \sin(\odot - l).\end{aligned}$$

$$\begin{aligned}\text{Aberration in longitude} &= -\frac{A'M}{\sin \Pi A} = -\frac{20''\cdot45 \sin OH}{\cos AH} \\ &= -20''\cdot45 \sec\lambda \cos(\odot - l).\end{aligned}$$

* The curve described by W is the hodograph of the earth's path, and therefore accurately a circle, but K is not the centre except on the supposition in the text—that the motion is uniform.

280. *To determine the aberration of a star in right ascension and declination.*

Let ΥOS be the ecliptic, ΥRT the equator, P its pole, S the sun, A the star, O the point 90° behind S as before.



From A', the apparent place of the star, draw $A'N$ at right angles to PAT. Join PO cutting the equator in R, and in AP produced make $AQ = 90°$, and join OQ.

Let ΥT the star's right ascension $= \text{Æ}$,

AT .... declination $= \delta$,

$O\Upsilon R$ the obliquity of the ecliptic $= \omega$.

Then $A'N = AA' \sin A = k \sin OA \sin A$

$$= k \sin OP \sin P$$

$$= k \cos OR \sin RT$$

$$= k \cos OR\,(\sin \text{Æ} \cos \Upsilon R - \cos \text{Æ} \sin \Upsilon R)$$

$$= k \sin \text{Æ} \cos \Upsilon O - k \cos \text{Æ} \cos \Upsilon \sin \Upsilon O;$$

and, aberration in right ascension $= - A'N \sec \delta$,

$$= - k \sec \delta \,\{\sin \text{Æ} \sin \odot + \cos \text{Æ} \cos \omega \cos \odot\}.$$

281. Again, aberration in declination $= - AN$

$$= - AA' \cos A'AN = k \sin OA \cos OAP = k \cos OQ$$

$$= k\,\{\cos OP \cos PQ - \sin OP \sin PQ \cos OPA\}$$

$$= k\,\{\sin OR \cos \delta - \cos OR \sin \delta \cos (\text{Æ} - \Upsilon R)\}$$

$$= k\,\{\sin \Upsilon O \sin \Upsilon \cos \delta$$

$$- \sin \delta\,(\cos \text{Æ} \cos OR \cos \Upsilon R + \sin \text{Æ} \cos OR \sin \Upsilon R)\}$$

$$= k\,\{\sin \Upsilon O \sin \Upsilon \cos \delta - \sin \delta\,(\cos \text{Æ} \cos \Upsilon O + \sin \text{Æ} \cos \Upsilon \sin \Upsilon O)\}$$

$$= - k\,\{\cos \odot \sin \omega \cos \delta + \sin \delta\,(\cos \text{Æ} \sin \odot - \sin \text{Æ} \cos \omega \cos \odot)\}.$$

282. It was a change in the polar distance of the star γ *Draconis* which first drew Bradley's attention to the subject, and ultimately led to the discovery of aberration.

This star was most favourably situated, because it passed so near to the observer's zenith that he had not to fear any errors of refraction. Moreover, its right ascension was nearly 270°, so that, at the time of the vernal equinox, when $\odot = 0°$, the aberration in declination is $-k \sin(\omega + \delta)$, and, at the time of the autumnal equinox, when $\odot = 180°$, it is $+k \sin(\omega + \delta)$.

These results follow immediately from the formula above, or they may be very simply deduced from a figure by means of the expression for aberration "k sin (earth's way)."

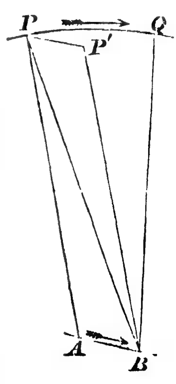
The polar distance of γ *Draconis* is therefore greatest about the end of March, and least about the end of September; the variation amounting to $2k \sin(\omega + \delta)$, or, since $\omega + \delta = 75°$ nearly, the change of declination is

$$40''\cdot 9 \sin 75° = 39''\cdot 5.$$

Solar, Lunar, and Planetary Aberration.

283. On account of the motion of the moon and of the planets, the light by which each is seen comes from a point of space which the body no longer occupies when its rays reach the observer.

Thus, if PQ be a portion of the actual path of a planet in space, Q its true place, and B that of the earth at the same moment, the planet will not be seen by rays coming from Q, but from a point P which the planet occupied at a time t before the observation; t being the time light takes to move from the planet to the observer.



The rays therefore reach the earth in the direction PB, but the earth's own velocity causes aberration, as in the case

of the stars, and if A be the position of the earth at the time t preceding the observation,

$$AB : PB = \text{vel. of earth} : \text{vel. of light};$$

therefore, completing the parallelogram $BAPP'$, BP' will be the apparent direction of the planet when its true direction is BQ. The angle between the two directions BQ and BP' is called the *planet's aberration.*

284. Since BP' is parallel to AP, and A and P were corresponding positions of the earth and planet at a time t preceding the positions B and Q, we have the following simple rule for determining *by calculation* the apparent place at a given instant, the true path being supposed known: find the time t which the light will take to reach the earth (the ratio of the planet's distance to that of the sun is supposed known), and calculate the true place for a time t preceding the given instant, this will be the apparent place at the given instant.

By reversing this rule, if we wish to find *by observation* the true place at a given instant, wait till a time t has elapsed and then observe the apparent place, this will be the true place required.

Thus, if $r = \rho a$ be the distance of the planet from the earth, where a is the mean distance of the sun, the light of the planet will take ρ ($8^m\ 18^s$) to reach the earth, and the apparent geocentric direction of the planet at any instant will be the direction it actually had ρ ($8^m\ 18^s$) previously.

In the case of the moon, $\rho = \frac{1}{400}$, and the aberration will always be very small (less than $0''\cdot5$).

The same rules will obviously apply to the sun; but, in this case, the body being fixed, Q will coincide with P, and all rays which reach the earth come from the point which the sun really occupies, and are therefore affected with stellar aberration only. The sun's true place in the ecliptic is always in advance of its apparent place, and we may calculate the amount of aberration as follows:

Let v be the velocity of the earth, and V that of light, θ the angle which the direction of the earth's motion makes with the radius vector r, then

$$\text{aberr.} = \frac{v}{V}\sin\theta = \frac{vr\sin\theta}{Vr} = \frac{\text{constant}}{r} \text{ (Newton, Sec. II. Prop. 1)}$$

$$\frac{20''\cdot 45}{\rho}, \text{ where } \rho \text{ is always near } 1,$$

$20''\cdot 45$ corresponds to the mean distance a.

Diurnal Aberration.

285. We have hitherto considered the velocity of the observer as being the same as that of the centre of the earth; there will, however, be in addition another aberration due to the rotation about the axis. A person at the equator will describe $2\pi.3960$ miles in a sidereal day. During that time the earth will be carried $\frac{2\pi.91533000}{366\frac{1}{4}}$ miles in its orbit. Therefore, if k' be the coefficient of diurnal aberration at the equator,

$$\frac{k'}{k} = \frac{3960 \times 366\frac{1}{4}}{91533000},$$

whence $$k' = 0''\cdot 324.$$

For an observer in latitude ϕ, we must use the coefficient of aberration $k' \cos\phi$.

The east point of the horizon is that towards which the observer is being carried, and which therefore will replace the point O of the previous general investigation (fig., p. 215).

The effect of diurnal aberration is so small that it may generally be neglected, except for a star very near the pole, whose right ascension, when on the meridian, will be increased by

$$0''\cdot 324 \cos\phi \sec\delta = 0^s\cdot 0216 \cos\phi \sec\delta.$$

CHAPTER XVIII.

PRECESSION AND NUTATION.

Precession.

286. WHEN catalogues of the true places of the stars, corrected for refraction and aberration, are formed at different epochs, as explained in Chap. I., Art. 16, it is found that the coordinates are all undergoing continual variations. Now, these variations may be due either to actual motions of the stars themselves, called their *proper motions*, or to a displacement of the planes and circles to which they are referred. The former cause would obviously produce special disturbances which would vary from one star to another, whereas any general displacement, affecting all the stars, must be attributed to the latter.

Long continued observations have shewn that the latitudes of all stars are *very nearly* constant, while their longitudes increase at a mean rate of $50''\cdot2$ per annum. From this we infer that the ecliptic is very nearly a fixed plane, and that the plane of the equator has a gliding retrograde motion which causes the first point of Aries to move backwards along the ecliptic at this mean rate of $50''\cdot2$ per annum.

When the right ascensions and declinations are examined in a similar manner, their variations are found to lead to the same conclusion—a gradual shifting of the plane of the equator; but we learn moreover from them that the inclination of the plane of the equator to that of the ecliptic, or what we have called the obliquity of the ecliptic, has an almost invariable value.

287. The investigations of physical astronomy shew that these results are accounted for by the action of the sun and moon on that portion of the earth's mass which, owing to its spheroidal shape, bulges out beyond the inscribed sphere. This attraction does not affect the mean value of the obliquity, but causes the line of the equinoxes to move backward on the fixed ecliptic, and thus to increase the longitudes of all the stars by a common quantity, the yearly value of which ($50''\cdot38$) is called the *luni-solar precession.*

288. The latitudes of the stars have been stated above to be only "very nearly" constant. They have, in fact, a general change which indicates a motion of the plane of the ecliptic itself; and here again physical astronomy furnishes an explanation, by shewing that the attractions of the planets tend to disturb the earth's path, that is, to alter the plane of its orbit, but have no effect on the plane of the equator. The effect is a diminution of the obliquity amounting to $48''$ in a century, and a slow progressive motion of the first point of ♈ along the equator, causing an annual decrease of the right ascensions of all the stars. This is called the *planetary precession.*

Effects of Precession.

289. The luni-solar precession and the planetary precession combined give the *general precession* $50''\cdot2$ yearly; but, in examining the effects of these changes, we shall neglect the planetary precession, and consider the plane of the ecliptic as fixed, and the $50''\cdot2$ as due to the motion of the equator alone.*

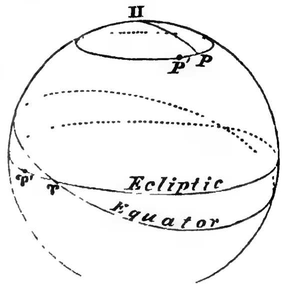
Let Π be the pole of the ecliptic,

P ... that of the equator,

♈ ... the point of intersection of the two circles.

* For a complete investigation we may refer to Chauvenet's *Astronomy*, vol. I., p. 605.

Precession, as we have said, carries ♈ backward along the ecliptic without altering the obliquity; but the obliquity is measured by ΠP the distance between the two poles, therefore the effect of precession is to carry the pole P backward along the small circle PP' parallel to the ecliptic. If ♈ be the position of the first point of Aries at the beginning of any tropical year, then ♈′ will be its position at the beginning of the next, ♈♈′ being $50''\cdot2$ measured backward, *i.e.* in a direction opposite to the sun's motion; and a complete revolution will be accomplished in 25,800 years. It is obvious that, by this regression, the tropical year is shortened, and the return of the equinox takes place earlier than it would otherwise have done, whence the term *precession of the equinoxes*, and the name *precession* applied to this motion.



Another effect of precession will be the gradual shifting of the constellations with respect to the equinoctial points (Art. 166). Hipparcus, about 120 B.C., discovered the precession and pointed out some of its necessary consequences. The first point of Aries was at that epoch in the constellation whose name it bears, but, though it has retained the name, it has shifted through nearly 30°, and is no longer in the constellation *Aries* but in *Pisces*. In the course of time, the stars seen at certain seasons will give place to others, our present winter constellations becoming summer ones, and *vice versâ*, until 25,800 years have elapsed, when the pole and the first point of Aries will return to their present positions.

The present polar star will, in about 13,000 years, be 46° from the pole; and, long before that, will have lost its claim to the title. At present it is still approaching the pole, and will do so yet for about 150 years; but the small circle PP',

which the pole describes, passes by other bright stars which will become the polar stars in turn.

Precession will also produce a slight alteration in the lengths of the seasons, as explained in Art. 181, but this effect is not so striking as those just described.

290. When the right ascension and declination of a star are known at any epoch, we can calculate its latitude and longitude (Art. 167). Adding $50''\cdot 2t$ to the longitude will give the new longitude after the lapse of t years, owing to precession. With this new longitude, and the same latitude and obliquity, we can obtain the new right ascension and declination. For a complete investigation of these changes we shall refer to Woodhouse's *Astronomy*, vol. I., or Chauvenet's *Astronomy*, vol. I., and shall here give only the precession in declination, obtained by differentiating the equation which connects it with the longitude.

If l be the longitude, λ the latitude of a star,

Æ...its right ascension, δ its declination,

and ω the obliquity, we have, from the triangle ΠPS, where S is a given star,

$$\sin\delta = \cos\omega \sin\lambda + \sin\omega \cos\lambda \sin l,$$

δ and l are the only variables, and differentiating with respect to t,

$$\cos\delta \frac{d\delta}{dt} = \sin\omega \cos\lambda \cos l \frac{dl}{dt}$$

$$= \sin\omega \cos Æ \cos\delta \frac{dl}{dt},$$

by formulæ (α) and (α'), Art. 167,

$$\frac{d\delta}{dt} = \sin\omega \cos Æ \frac{dl}{dt};$$

therefore the annual precession in declination

$$= 50''\cdot 2 \sin\omega \cos Æ.$$

Nutation.

291. In treating of precession we stated that $50''\cdot2$ was the *mean* value of the yearly motion of ♈, and that the obliquity of the ecliptic was very nearly constant. Bradley, when discussing his observations after the discovery of aberration, found that the changes of declination of the stars could not all be accounted for by precession and aberration alone. His results led him to infer that there exists a small subordinate motion, by which the pole of the equator is carried sometimes before, and sometimes behind, the mean place to which a uniform motion in the small circle would have brought it; at the same time the distance from the pole of the ecliptic is sometimes more, and sometimes less, than the mean value; so that the *true* path of the pole is of a wavy form. He also found that these changes were periodic, and that they completed their cycle in about 19 years.

Bradley found an intimate connection between these oscillations of the earth's axis, to which he gave the name of *Nutation*, and the inclination of the moon's orbit to the plane of the earth's equator. This inclination varies with the position of the moon's node, *i.e.* of the line where the plane of the moon's orbit crosses the ecliptic, and this line describes a complete revolution in 18 years 220 days. The researches of physical astronomy have fully confirmed Bradley's suggestion, that the cause of nutation is to be found in the variable action of the moon in causing precession.*

The complicated motion of the pole of the equator may be expressed as follows:—Suppose pp' to be the small circle described by the mean pole p. About p as centre, describe an ellipse with a major axis $18''\cdot5$ directed towards Π the pole of the ecliptic, and a minor axis $13''\cdot7$ on the small circle. Then, as the *mean* pole p moves uniformly along

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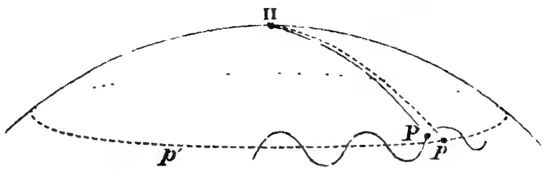
* See Airy's *Tracts*, "Precession and Nutation."

the small circle in a retrograde direction, carrying the ellipse with it, the *true* pole P will move along the circumference



of the ellipse, completing a revolution in 18 years 220 days.

The *true* first point of Aries being the pole of ΠP, will differ from its *mean* position which is the pole of Πp, except when P is at the extremities of the major axis of the moving ellipse. The difference between the true and the mean position of ♈ is called the *equation of the equinoxes.*

292. We cannot better conclude this chapter than with an extract from Woodhouse's *Astronomy*, giving an account of the means by which Bradley separated nutation from aberration:

"The star γ *Draconis*, passing the meridian very near the zenith of Bradley's observatory, and being consequently very little affected by refraction, was the chief star of his observations. This star, in March, passed more to the south of the zenith by about 39″ than it did in September. .....Other stars also changed their declinations. The changes of declination of a small star in *Camelopardalus*, with an opposite right ascension to that of γ *Draconis*, were observed at the same time as those of the latter star; and, it was Bradley's argument, that, if these phenomena (changes of declination) arose from a real nutation of the earth's axis, the pole must have moved as much towards γ *Draconis* as from the star in *Camelopardalus;* but this not being the case, the hypothesis of a nutation of the earth's axis would not account for the observed phenomenon: more strictly speak-

ing, it would not completely account for it, for, in fact, some part of the observed changes of declination was due to the effect of nutation.

"Bradley, as we have seen, solved the above phenomena by the theory of aberration. Now, if such theory, with the known one of precession, would account for all observed change of zenith distances, or, of north polar distances, then there could be no changes but what arose from precession and aberration. Hence, since the aberration is the same at the same season of the year, the distance of γ *Draconis*, in September, 1728, ought to have differed from its distance, in September, 1727, only by the annual precession in north polar distance; the distance, in September, 1729, from the distance in September, 1727, by twice the annual precession in north polar distance; and so on. Such, however, was not the observed fact. In 1728, after the effect of precession had been allowed for, γ *Draconis* was nearer the north by about $0''\cdot8$ than in 1727. In 1729, nearer than in 1727, by $1''\cdot5$. In 1730, by $4''\cdot5$. In 1731, by nearly $8''$. Here then was a new phenomenon, a change of north polar distance, indicating an inequality not yet discovered.

"Bradley observed other stars besides γ *Draconis;* amongst others, the small star above mentioned of *Camelopardalus;* and, it is not a little worthy of notice, this same star, which, in the case of the former inequality (that of aberration), directed him to reject the hypothesis of a nutation of the earth's axis, here determined him to adopt it. For, within the same periods, the changes in north polar distance of γ *Draconis* and of the star in *Camelopardalus*, were equal and in contrary directions; that is, whilst the former, through the years 1728, 1729, 1730, 1731, was approaching the zenith, and consequently the pole, the latter was, by equal steps, receding from the zenith, and consequently from the pole. These phenomena then of the changes in the north polar distances could adequately be explained by supposing a

nutation in the earth's axis *towards* γ *Draconis*, and *from* the small star in *Camelopardalus*.

"After 1731, Bradley observed contrary effects to happen; that is, γ *Draconis* receded from the zenith and north pole, and the star in *Camelopardalus*, by equal steps, approached those points; and this continued till 1741 (a period of more than nine years); after which the former star again began to approach the zenith, and the latter to recede from it. These phenomena then, between 1731 and 1741, could be adequately explained by supposing, during that term, a nutation in the earth's axis, *from* γ *Draconis* and *towards* the small star in *Camelopardalus*.

* * * * *

"By examining various and numerous observations, and by discriminating those that happened at particular conjunctures, Bradley found abundant confirmation of the truth of his two theories—aberration and nutation. During a period of more than twenty years, he accounted for the phenomena of observation, that is, the changes in the declinations of various stars, by making those changes or variations consist of three parts—the first due to precession, the second to aberration, and the third to nutation; the quantities and laws of the two latter being assigned on the principles and by the formulæ of his theories.

"We cannot sufficiently admire the patience, the sagacity, and the genius of this astronomer, who, from a previously unobserved variation not amounting to more than 40 seconds, extricated, and reduced to form and regularity, two curious and beautiful theories."

CHAPTER XIX.

THE MOON.

293. NEXT to the sun, the moon is, to the inhabitant of the earth, the most important of the heavenly bodies. Its size, its rapid motion among the stars, its influences both real and supposed, must, from the earliest times, have given it a prominent place in the observations of astronomers.

Like the sun, it advances among the stars in a direction opposite to that of the diurnal motion, but about thirteen times faster; a complete revolution being performed with respect to

the fixed stars in.........	27 d.	7 h.	43 m.	11·461 s.*
... first point of ♈	27 d.	7 h.	43 m.	4·614 s.
... sun....................	29 d.	12 h.	44 m.	2·87 s.

The difference between the first and third of these periods is due to the advance of the sun itself. During the 27⅓ days of the moon's sidereal revolution, the sun will have moved through some 27°, which it will take the moon about 2 days 5 hours to gain.

The difference between the first and second periods is accounted for by the small regress of the first point of ♈ in 27 days.

* These are the values at present, for, comparison with ancient observations led Halley to the conclusion that the moon's mean velocity is being accelerated, and the period of a revolution shortened. La Place proved theoretically that this acceleration is confined within very narrow limits, and will be followed by a retardation.

294. The period of a revolution with respect to the sun is called a *lunar month* or a *lunation*, and also a *synodical* period; and the commencement of the period is the instant when the bodies have the same longitude. It is obvious that they have not necessarily the same right ascension at that instant.

When two bodies have the same longitude they are said to be in *conjunction*, when their longitudes differ by 180° they are in *opposition*, and when by 90° they are in *quadrature*. The points distant 45° from these four positions are called *octants*. The two positions 'conjunction' and 'opposition', when spoken of jointly, are called *syzygies*.

295. If the apparent diameter of the moon be measured at different times, it will be found to vary within certain narrow limits; its distance from the earth will therefore also vary in a corresponding, but inverse, manner. Observations of the parallax (Art. 262) shew that the mean distance is about 238,000 miles, or 60 times the earth's radius, and that the variation is about $\frac{1}{20}$ of this mean value, making the distance sometimes 57 radii, and sometimes 63.

The moon may, therefore, be considered as a companion of the earth in its orbit round the sun, and is, in fact, called the earth's satellite.

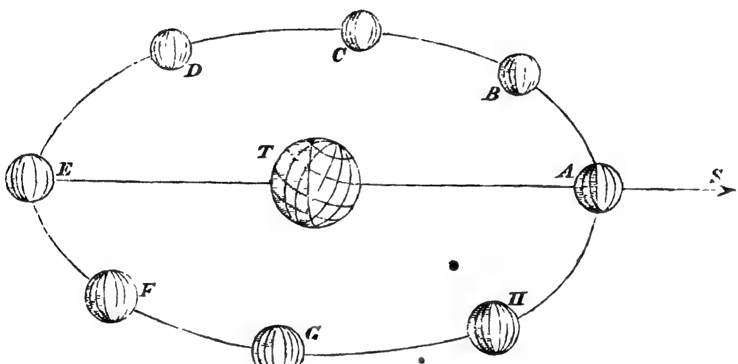
296. One of the most striking phenomena connected with the moon is the change of its visible outline, or its *phases*, as the successive appearances are called.

The moon being an opaque spherical body, which receives its light from the sun, becomes visible to us by reflecting this light. It is obvious that the hemisphere, which is turned towards the observer, will not generally be all lighted up, and that consequently the appearance presented will seldom be a complete circular disc, but will vary with the relative positions of the sun, the moon, and the observer.

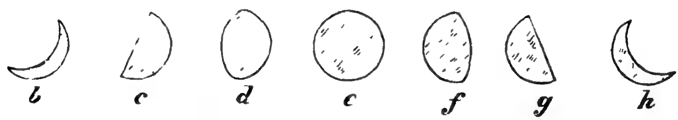
The phenomenon of the phases will be understood from

the accompanying diagram, which represents one synodical revolution of the moon about the earth, the sun being supposed to remain stationary in the direction *TAS*. The distance of the sun from the moon being so great, in comparison with the dimensions of the bodies themselves, we may consider that one-half of the moon is always illuminated, and the other half, dark.

When the moon is in conjunction at *A*, the dark side



is turned towards the earth, and no portion of it is then visible. It is *new-moon*.



About a week later, the moon is at *C*, 90° from *A*; the plane of separation of its bright and dark parts passes through the earth, and the visible portion appears like a bright semi-circular disc as *c*. This is the *first quarter*.

In intermediate positions, such as *B*, the illuminated portion appears as a crescent (*b*), gradually expanding into the semi-circular form at *C*.

After passing *C*, a greater portion of the illuminated half becomes visible, and the bright disc swells out at the centre, and presents the appearance (*d*) which is called *gibbous*.

About $14\frac{1}{2}$ days after conjunction, the moon will be in opposition at *E*, and, the illuminated face being wholly turned towards the earth, presents a complete circular disc (*e*) called *full-moon*. From full-moon the sequence of changes will be precisely the same, but in a reverse order; the disc being gibbous at *F*, half-full at the third quarter *G*, less than half at *H*, then disappearing altogether at the next new-moon.

297. During the first half of the month, the moon will be less than 180° east of the sun, and will cross the meridian between noon and midnight. The western limb is the bright limb during that period, as figured in the diagram, from *b* to *e*. In the second half, the moon will cross the meridian between midnight and the next noon, and the eastern limb becomes the bright limb (figs. *e* to *h*).

Although, strictly speaking, the crescent is formed as soon as the moon leaves *A*, it only becomes visible when at an angular distance of 30° or 40° from *A*; the thin line of light which it presents being overpowered by the strong light of the sun.

For the recurrence of the moon's phases, in accordance with the days of the calendar month, see the explanation of the golden number, Chap. XXIII.

298. A simple inspection of the figure will shew that the earth must present phases to the moon—the exact counterpart of those which the moon presents to the earth. Thus, when it is new-moon, the illuminated side of the earth will be towards the moon, which will then have full-earth. When the moon is a crescent at *B*, the earth will appear gibbous, and so on.

This will explain a phenomenon which is often observed: After sunset, when the moon is a crescent, the remainder of the circular disc is frequently visible, shining with a pale

grey light.* This faint light is due to the strong *earth-light* which then falls on the moon, and which the moon reflects back again to the earth.

As the moon increases its angular distance from the sun, the amount of earth-light received by it diminishes, and the effect disappears.

299. The line joining the two *cusps* is a diameter of the circle which separates the dark from the bright hemisphere; it is therefore perpendicular to the line which joins the centres of the sun and moon. Again, it is a diameter of the circle which separates the hemisphere turned towards the observer from the opposite one, and is therefore perpendicular to the line joining the observer with the centre of the moon. It is therefore perpendicular to the plane which passes through the observer and the centres of the sun and moon; or, in other words, the great circle joining the centres of the sun and moon will bisect the line of cusps at right angles.

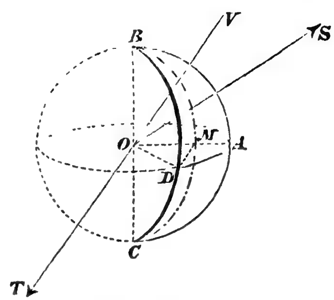
The angle this great circle makes with the horizon is very variable, so that in corresponding positions of the crescent moon in different months the line joining the cusps will be very differently inclined, being sometimes nearly horizontal, and at others nearly vertical.

300. The bright portion of the visible hemisphere of the moon is bounded by two semicircles; but as the inner one is seen obliquely, it is projected into a semi-ellipse, whose major axis is the diameter of the moon, and whose minor axis is constantly varying. The following investigation will

* Called 'lumière cendrée' by the French. The faintly luminous portion of the moon will appear as if belonging to a smaller sphere than the bright crescent does; but this is an illusion due to *irradiation*. The phenomenon itself is sometimes popularly spoken of as 'The old moon in the arms of the new.'

shew how the magnitude of the phase is connected with the positions of the sun and moon.

Let O be the centre of the moon, OT the direction of the observer, BAC the plane through O perpendicular to this direction; OS the direction of the sun, and BDC the plane perpendicular to OS; then the lune comprised between BAC and BDC constitutes the visible luminous portion; and if BDC be projected orthogonally on the plane of BAC, the projection BMC will be the semi-ellipse forming the inner boundary of the luminous disc.



Now $$OM = OD \cos DOM$$
$$= OA \cos SOV,$$

where OV is the prolongation of TO; therefore

$$AM = OA - OM$$
$$= OA \text{ versine } SOV.$$

The area of the illuminated disc varies as AM, *i.e.* as the versine of the exterior angle of elongation SOV.

301. When $SOV = 90°$, BMC becomes a straight line, and conversely. Hence, if the angular distance OTS between the sun and moon, as seen by the observer T, be measured at the moment when the moon's disc is just half illuminated, two angles of the triangle OST will be known, and thence the ratio of ST to OT can be found, and the parallax of the sun determined in terms of that of the moon.

This method of finding the solar parallax is not practically available, on account of the difficulty—we may say impossibility—of fixing upon the precise instant when the moon is dichotomised, as this phase is sometimes

called.* The regularity of the inner boundary is disturbed by the lunar mountains, and, when examined with a telescope, the line is found broken and jagged, with detached points of light here and there encroaching on the darker part; these detached points are the mountain tops which receive the sun's light while the intervening valleys are still in the shade.

302. *Age of the Moon.* We have said that new-moon is the instant when the centres of the sun and moon are in conjunction. The age of the moon, at any instant, is the time, expressed in days, that has elapsed since the previous new-moon. When only integral values are employed, the moon is said to be one day old when less than 24 hours have elapsed since new-moon, two days old during the next 24 hours, and so on. The moon's age is given, in the Nautical Almanac, to the nearest tenth of a day for each Greenwich noon (see Chap. XXIII.).

Moon's Orbit. Nodes.

303. When observations are carried on for a long period in order to ascertain the path of the moon, as was done in the case of the sun (Chap. VII.), it is found that its motion is much more complex. The right ascensions and declinations being determined day by day, and the positions marked on the globe, it is found that the curve described during each revolution, though approximately, is not accurately, a great circle, nor even a plane curve.

We shall refer the motion to the ecliptic, the results being simpler than when referred to the equator:—If at any instant a great circle be drawn through the direction of the moon's motion, this great circle will intersect the

* Aristarchus employing this method, found that the line of separation was a straight line when the moon was 87° from the sun, whence he inferred that the sun's distance from the earth was 19 times greater than the moon's, instead of 400 times, which modern observations, by more correct methods, have given.

ecliptic in two opposite points called the moon's nodes; the *ascending-node* being that where the moon crosses from the south to the north side of the ecliptic, and the other the descending node. The observations shew that these points are not stationary, but that, while the plane of the orbit itself remains inclined at a constant* angle of about 5° 9′ to the plane of the ecliptic, the nodes regrede along the ecliptic at an average rate of 3′ 10″·6 per day, or 1° 27′ in each sidereal revolution of the moon. In one year the node is carried about 19° 20′ back, and in about 18·6 years returns to its first position.

The greatest latitude is therefore always 5° 9′ in every revolution; but the greatest declination will vary, according to the position of the line of nodes, from 18° 18′, when the plane of the orbit lies between the equator and ecliptic, to 28° 36′ when outside; the first value being the difference, and the second the sum, of the obliquity of the ecliptic and the inclination of the orbit to the ecliptic.

304. The variation of the moon's apparent diameter indicates changes in her distance from us. If we pursue the same method of observation as for the sun (Chap. XI.) we shall find, (disregarding the shifting of the plane of her orbit, which during one revoltion is less than 1½°) that the moon describes an ellipse, the earth occupying one of the foci.

The excentricity of the elliptic orbit is about $\frac{1}{20}$; the greatest and least distances being respectively 251,700 miles and 225,600 miles.

When the moon is at her greatest distance, she is said to be in *apogee*, and when at her least distance in *perigee*. Those points of the orbit are jointly spoken of as the apses, and the line joining them the line of apsides.

305. We found in the case of the sun (Art. 182) that the apses advance along the ecliptic. The moon's apses also

* Approximately constant.—there are small periodical fluctuations.

progrede, but much more rapidly, nearly 40° in one year, or 3° in each revolution, so that in about $4\frac{1}{2}$ years the perigee arrives where the apogee was before.

There are many other fluctuations and disturbances to which the moon is subject. The largest of these were known to ancient astronomers, and the most important of the small inequalities had been detected by observation before their physical cause was known. A complete account of them, however, is beyond the scope of the present work, and the investigation of all the perturbations can only be effected by the most refined analysis of physical astronomy.

Librations.

306. The moon always presents the same, or very nearly the same, face to an observer; the mountains and valleys which cover the surface of our satellite are seen occupying nearly constant positions relatively to the centre of the disc, and relatively also to the plane of the orbit. We infer, therefore, that the moon revolves about an axis nearly perpendicular to this plane, that the time of a rotation about this axis and of a revolution round the earth must be very nearly equal, and that the average of a large number of these periods must be exactly the same for both.*

* A controversy seems to be periodically arising as to the proper words to be used in describing this motion of the moon. There is no question about the phenomenon itself: both parties understand clearly what the character of the motion is, and would probably, if asked to represent it by a model, make use of the very same contrivance; but the one asserts that (neglecting librations and deviations from a circular path) the motion consists of rotation round the earth's axis only, while the other describes it, as is done in the text, as a rotation about its own axis combined with a revolution round the earth.

In the supposed case of circular and uniform motion, both modes of expression are correct; but the mathematician would adopt the latter form in preference to the former, because the usual—and often the only practicable—way of investigating the motion of a free rigid body is by determining, firstly, the motion of its centre of gravity, and next, the motion of certain lines fixed in the body and moving with it. It is a curious fact that the equations for determining these two parts of the motion are independent, and this separation of the two sets of equations becomes so familiar, that the mind may bring itself

The angular velocity of the moon about its own axis is uniform, but its angular velocity about the earth is not so; hence we shall sometimes see a little more of the eastern limb, sometimes a little more of the western. This constitutes the phenomenon called *libration in longitude.*

Again, the axis about which the moon rotates is not quite perpendicular to the plane of her orbit, though very nearly so, and hence a little beyond each pole will come alternately into view. This is the *libration in latitude.*

A third libration is the *diurnal* or *parallactic libration* which arises from parallax. When we said that the moon always presents the same face to the observer, it was understood that the observer was supposed at the centre of the earth. It is obvious that at the rising and setting of the moon, portions become visible beyond the upper limb which disappear as the moon's altitude increases.

307. To the inhabitant of the moon, if any, the days and nights will be approximately equal, and each about $14\frac{3}{4}$ of our days. The earth is, however, a moon to the moon, and will, to one half of our satellite, present the remarkable appearance of a globe, about 2° in diameter, fixed (except for

to look upon the corresponding physical facts—the translation of the centre of gravity and the rotation about an axis through the centre of gravity, as the only natural expressions of the motion.

But we may also proceed in a different manner: we may determine the motion of the instantaneous axis, and then the angular velocity about that axis. Now, when the instantaneous axis is fixed, this is perhaps the simplest way of conceiving the motion; and it is the view taken by those who hold that the moon rotates about the earth and has no other motion. For, if we neglect librations and deviations from a circular path, the instantaneous axis will be a fixed axis passing through the earth's centre.

When, therefore, a discussion has arisen, the error committed by both parties has been that of denying any other conception of the motion to be possible besides their own. Each, finding that his own idea would perfectly explain the phenomenon, has taken it for granted that every other must be wrong.

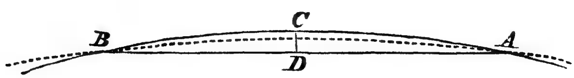
When the librations and departure from circular motion are taken into account, the actual phenomena can no longer be represented by a rotation about the earth's axis; whereas, they are still accurately shewn by the other mode of representation.

small librations) in the sky, and having phases like those of the moon, but, on account of its size, giving about 13 times more light.

Path of the Moon round the Sun.

308. That motion of the moon, which we have hitherto considered, is relative to the earth, and would be her absolute motion in space if the earth were fixed; but, as the two bodies move together round the sun, the actual path of our satellite, relatively to the sun, will be due to a combination of her own monthly motion round the earth, and of the earth's yearly motion about the sun.

The orbit of the earth is elliptical, almost circular, and as there are about $12\frac{1}{2}$ lunations in a year, the moon's path must—neglecting the small inclination of the two orbits—cross that of the earth about 25 times. We might, perhaps, from this be led to expect that the curve described by the moon would consist of a series of loops or waves, as in figs. (a) and (b). Such, however, is not the case. *The moon's orbit is everywhere concave to the sun.* The strict investigations of physical astronomy are necessary to prove the correctness of this statement,* but the following calculation will shew its probability: Let the curve ACB represent



a part of the earth's orbit, A a point at which it is crossed by the moon when coming within, and B the next intersection

* See the Author's *Elementary Treatise on the Lunar Theory*, 3rd Edit., p. 85.

when going outside. If the moon's path is anywhere convex, it should be between A and B.

Draw the chord AB and the sagitta CD. AB is about $\frac{1}{25}$ of the orbit, and therefore subtends an angle $\frac{2\pi}{25}$ at the sun; and, if R be the radius of the earth's orbit, we have approximately,

$$CD = R\left(1 - \cos\frac{\pi}{25}\right)$$

$$= \tfrac{1}{2}\left(\frac{\pi}{25}\right)^2 R$$

$$= \tfrac{3}{400}R \text{ nearly}$$

$$= 3 \text{ times the moon's distance from the earth.}$$

The moon's path, therefore, lies between the chord ADB and the arc ACB, and we may conclude, with great probability, that it is everywhere concave to the sun, as represented by the dotted line.

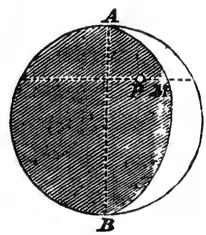
Lunar Mountains.

309. To Galileo is due the discovery that the surface of the moon is covered with mountains and valleys. He also concluded from his observations that several of the mountains rise to an altitude of between four and five miles above the surrounding plane, and his results have been confirmed by the researches of Messrs. Beer and Mädler (1837). As the moon's diameter is only $\frac{3}{11}$ of that of the earth, we see that her mountains are comparatively very much loftier.

When viewed through a telescope, the lunar mountains project shadows on the side opposite to the sun, and the lengths of these shadows, when measured by a microscope, will serve to determine the heights of the mountains above the planes on which the shadows are cast, proper account being taken of the inclination of the sun's rays. This is the method pursued by Messrs. Beer and Mädler.

310. Another method, which we shall proceed to describe, consists in measuring the distance of the small detached points of light which sometimes appear on the dark portion of the moon's disc, from the line of separation of light and shade. These points of light are the mountain tops, which catch the sun's beams over the edge of the spherical surface while the intermediate plane is still in shade.

Let the figure represent the visible disc of the moon, P the projection of the bright summit of a mountain peak seen on the dark portion at the time when the sun's light just reaches it.



The distance of P from the edge M must be measured with a micrometer in a direction PM perpendicular to the line of cusps, and the distance AB between the cusps must also be measured. Let m be the ratio of these measures.

Then PM is the projection of a tangent to the sphere, parallel to the sun's rays, and therefore making with PM an angle the complement of the exterior angle of elongation SOV of figure, p. 234.

Let $PM = c$, and $AB = 2r$, and let the height of the mountain $= x$.

Then $x(2r + x) =$ sq. of tangent

$=$ sq. of line of which PM is the projection

$= c^2 \operatorname{cosec}^2 O$,

where O is the exterior angle of elongation; therefore

$$\frac{x}{2r} = m^2 \operatorname{cosec}^2 O, \text{ neglecting the fraction } \left(\frac{x}{2r}\right)^2.$$

Whence the height of the mountain will be found, since r, m, and O are known.

Harvest Moon.

311. The moon crosses the meridian about $50\frac{1}{2}$ minutes later every day. This value is obtained as follows: In one synodical revolution the sun crosses the meridian exactly once oftener than the moon, therefore $29\frac{1}{2}$ solar days are equivalent to $28\frac{1}{2}$ lunar days, and the average length of a lunar day

$$= \frac{29\frac{1}{2}}{28\frac{1}{2}} \text{ solar days} = 1^{d}\ 0^{h}\ 50\tfrac{1}{2}^{m}.$$

If the moon moved along the equator, and at a uniform rate, we should find that the times of rising or setting would get daily later by precisely $50\frac{1}{2}^{m}$; but its path nearly coincides with the ecliptic, and this fact causes considerable variations in the daily retardation. These variations depend on the latitude of the place: at Cambridge the retardation may amount to $1^{h}\ 15^{m}$, and at other times be only 18 or 20 minutes.

Now, it has been observed, that at the full moon nearest to the autumnal equinox, the times of rising on three or four successive evenings will follow sun-set at a small interval; and the farmers, "not doubting that it had been so ordered on purpose to give them an immediate supply of moon-light for their greater conveniency in reaping the fruits of the earth,"* gave the name of harvest moon to this particular full moon.

To explain this phenomenon, let us remark, that if a body approaches the elevated pole without altering its right ascension, its stay above the horizon is increased, but the sidereal time of crossing the meridian is unaltered; therefore the time of rising is accelerated, and the acceleration will be greater, the greater the change of declination.

If, on the other hand, the right ascension be increased

* Ferguson's *Astronomy*.

without any change of declination, the time of rising will be retarded in proportion to the increase of right ascension.

When both circumstances exist, the retardation due to the increase of right ascension will be modified by the change of declination.

For simplicity, we shall suppose the moon's orbit to coincide with the ecliptic. Then supposing its daily motion in the orbit to be uniform (about $13\frac{1}{2}°$), it will be easily seen, that the change of declination is most rapid when crossing the equator, and that at the same time the change of right ascension is slowest. When 90° from these points, the declination changes very slowly, and the right ascension increases by considerably more than $13\frac{1}{2}°$ in one day, since the arc described being nearer the pole will subtend an enlarged angle.

In northern latitudes it will therefore be when the moon crosses ♈ that the conditions for a small retardation will arise; and, as the moon passes through this point at each revolution, the phenomenon of a small retardation must recur every month. It is, however, only at the autumnal equinox that the effect is noticed; for, the sun being then in ♎, the moon, when in ♈, will be full, and the rising take place near sunset. In other months the moon, when in ♈, is only partially illuminated, and rises either during the day or late at night. After two or three days the increase of declination is slower, and that of right ascension faster, and the daily retardation soon increases till it attains, and then exceeds, its mean value.

In southern latitudes the same phenomena will take place at the other equinox, which also will correspond with their harvest.

If we take the actual orbit of the moon, which is inclined 5° to the ecliptic, the effect will be greater or less in different years, according as the position of the nodes makes the angle between the orbit and the equator greater or less, the cycle being completed in 18·6 years (Art. 303).

The magnitude and the general character of a solar eclipse vary with the position of the observer, because the moon's parallax being large (nearly 1°), and the sun's small, a change of place on the earth will alter the apparent place of the moon and scarcely affect the sun's. Thus, the sun may be totally or annularly eclipsed to one observer, and only partially so to another, while, at the very same time, a large portion of the earth will have no eclipse at all. An eclipse may even be total to one observer and annular to another:—this would happen if, during the progress of the eclipse, the two apparent diameters became exactly equal; then, two observers, to each of whom the eclipse happened to be central, the one before and the other after this instant of equality of the diameters, would have, the one a total, and the other an annular, eclipse.

Lunar Eclipse.

315. Since we have no celestial neighbour nearer than the moon, it is obvious that an eclipse of the moon cannot arise from the same cause as an eclipse of the sun. But the moon's light is derived from the sun, and when the earth interposes itself between the two, it cuts off the sun-light, and thus a part, or the whole, of the moon becomes dark, and a partial, or total, eclipse of the moon takes place.

The eclipse cannot be annular; for, the cone of shadow projected by the earth is always, where the moon crosses, about three times as broad as the moon itself. The appearance of the edge of the shadow on the disc is an arc of this circular section of the cone.

An eclipse of the moon can only take place at, or near, opposition, that is, at full moon; but the latitude of the moon may be such as to enable it to pass the shadow without entering, which explains why there is not a lunar eclipse at every full moon. As in the case of solar eclipses,

a lunar eclipse will occur only when the moon, being full, is near a node of its orbit.

316. An essential distinction between the solar and the lunar eclipses is this:—in the one, the luminary is merely hidden from us; in the other, it actually loses its light. So that, as stated above, the character of a solar eclipse will vary from one observer to another, whereas in a lunar eclipse all parts of that hemisphere of the earth which is turned towards the moon will see the eclipse, and in precisely the same phase.*

Ecliptic Limits.

317. It is found that, in order that a *solar* eclipse may be possible, the angular distance of the sun's centre from the node, at the instant of conjunction, must not exceed 18° 36′; and that an eclipse will certainly happen if this distance be less than 13° 42′. These are called the solar ecliptic limits. Between these values the eclipse is doubtful.

It is also found that there will certainly be a *lunar* eclipse, provided the distance from the node, at the moment of full moon, be less than 9°; and that the eclipse will be impossible if this distance exceed $12\frac{1}{2}°$. These are the lunar ecliptic limits.

Synodic Revolution of the Moon's Node.

318. The moon's node has a daily retrograde motion of 3′ 10″·64, and the mean motion of the sun is 59′ 8″·33. The relative motion is therefore 62′ 19″ daily, and the sun will return to the same node in

$$\frac{360 \times 60}{62\frac{19}{60}} = 346{\cdot}62 \text{ days.}$$

* The parallactic libration will produce a small change, but the want of definiteness about the edge of the shadow renders it unnecessary to consider this.

Hence, 173 days after passing through one node, the sun will come to the other.

If the line of nodes retained a fixed position in the ecliptic, the sun would return to it year after year at the same or at very slowly changing dates, and the eclipses of the sun and moon would recur in the same months for a very long period. The retrograde motion of the nodes hastens the return of the eclipses and causes a constant and rapid shifting of their dates, a complete circuit of the calendar taking place in about 18 years (Art. 303).

Number of eclipses at one Node.

319. In $14\frac{3}{4}$ days, which is the interval between new-moon and full moon, the sun and node will separate by $14\frac{3}{4}\,(62'\ 19'') = 15\frac{1}{3}°$.

If, then, a full-moon happen exactly at the node, the preceding and the following new-moons will happen at $15\frac{1}{3}°$ from it, and therefore within the superior limits of a solar eclipse, so that three eclipses may occur at that node—two solar and one lunar.

But, if a new-moon occur exactly at the node, the preceding and the following full-moons will be beyond the lunar ecliptic limits, and only one eclipse (a solar one) will take place at that node.

The same results will follow if a full-moon or a new-moon happen, not exactly at the node, but within a couple of days on either side of it; the preceding and the succeeding syzygy will both be less than $18\frac{1}{2}°$, and both more than $12\frac{1}{2}°$ from the node.

Hence, at every passage of the sun through a node, there will be at least one eclipse, and there may be three.

Number of Eclipses in a Year.

320. The sun takes 173 days to pass from one node to the other, and six lunations occupy 177 days; so that a lunar

eclipse happening exactly at one node will give a lunar eclipse 4 days after the sun passes the next node, and therefore too far from it to produce three eclipses at that second node.

If, however, the lunar eclipse at the first node happens two days before the sun reaches it, the lunar eclipse at the next node will be two days only after the sun has passed it, and there may be three eclipses at each of the nodes. The sun going on will meet the first node again, and another lunar eclipse will occur six days after passing through; this second passage through the first node can, however, produce only two eclipses, viz. the lunar eclipse just spoken of and a solar eclipse at the preceding new moon. This solar eclipse occurs 12 lunations later than the first solar eclipse of the two groups of three, and 12 lunations occupy 354 days, so that these *seven* eclipses may all be comprised in the same year, provided the first of the seven occur early in January.

$12\frac{1}{2}$ lunations occupy $368\frac{3}{4}$ days, and therefore the eighth eclipse, the lunar, cannot come in. In order to bring it in, it would be necessary to shift the whole system back some days, but then the first solar eclipse of the first group would find itself in the December of the previous year, and the number of eclipses would still be *seven*.

Therefore, *in one year there cannot be more than seven eclipses*, five of the sun and two of the moon, or four of the sun and three of the moon.

In a similar manner it may be shewn that a single (solar) eclipse near one node may be followed by a single (solar) eclipse near the next node.

Therefore, *there cannot be fewer than two eclipses every year*, both of the sun.

321. Speaking generally, there are more eclipses of the sun than of the moon; thus, in a period of eighteen years there are on an average seventy eclipses—forty-one of the

sun and twenty-nine of the moon—that is, roughly, in the same ratio as their ecliptic limits.

But more eclipses of the moon than of the sun are seen at any given place, for the reasons already stated, that a lunar eclipse is visible to a whole terrestrial hemisphere at once, whereas a solar eclipse is visible only to a small portion of it.

Total or annular eclipses of the sun are phenomena of very rare occurrence in a *given* locality, although there are on an average twenty-eight for the whole earth in every period of eighteen years. In London, according to Halley, no total eclipse has been observed between the 20th of March, 1140, and the 22nd of April, 1715, a period of 575 years. The next total eclipse, visible in England, will be on the 19th of August, 1887.

The Saros of the Chaldeans.

322. The ancients, who had no correct tables of the sun and moon to enable them to predict the eclipses with the precision to which modern astronomy has arrived, had nevertheless discovered a method of extreme simplicity, by the use of which they foretold these phenomena with very considerable accuracy. This method is still used to determine at what new-moons, or full-moons, eclipses will occur—the strictly accurate modern methods being afterwards employed to calculate the character and details.

From what has been stated in the previous articles, we infer, that when the sun, the moon, and the node, return to the same relative positions, the same eclipses must recur.

Now	1 lunation occupies	29·53059 days,
and	1 synod. revol. of node	346·62 days;
therefore	223 lunations occupy	6585·32 days,
and	19 synod. revol. of node	6585·78 days.

Hence after 223 lunations, that is, a period of 18 years 11 days, or 18 years 10 days, according as 4 or 5 leap years

are comprised, the sun, the moon, and the node will return approximately to the same relative positions.

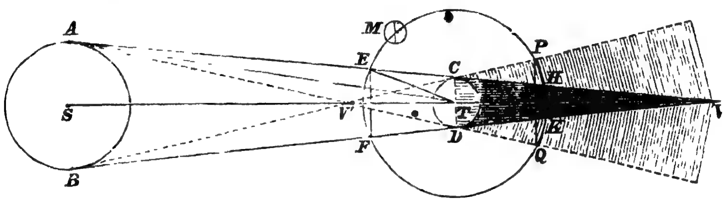
If, therefore, during one of these cycles of 18 years 11 days, a record be made of all the eclipses which occur, they will be found approximately to repeat themselves.

This period was known to the Chaldeans and called *Saros.*

Conditions necessary for a Solar or Lunar Eclipse.

323. We shall proceed to investigate the circumstances of solar and lunar eclipses in a more particular manner.

Let AVB be a cone which envelopes both the sun S and the earth T, the vertex V being beyond the earth; then the portion between T and V will receive no light from the sun.



About T as centre, describe a sphere passing through the centre M of the moon and cutting the cone in two circles whose diameters are EF and HK.

There will be an eclipse of the sun at *some* place on the earth if any portion of the moon come within EF; and there will be an eclipse of the moon if it enter HK.

The figure shews that EF is greater than HK, and would lead us to expect, as we have already seen, a greater number of solar than of lunar eclipses.

Let a second cone envelope both the sun and earth, but on opposite sides, having its vertex V' between the two bodies, and let it cut the sphere about T on the further side in a circle whose diameter is PQ; then, as soon as the moon enters PQ, it will receive light from a portion only of the

sun's surface. The space comprised between the two cones PQ and HK is called the *penumbra*, the dark part CVD being the *umbra* or *shadow*.

When the edge of the moon enters the penumbra, it is not eclipsed, but its light begins to diminish until it reaches the shadow, and the diminution is so gradual, that it is extremely difficult to ascertain by observation the exact instant of the commencement of the eclipse.

324. A solar eclipse will take place at, or near, conjunction, if the angular distance between the centres of the sun and moon, as seen from the centre of the earth, is less than the semi-diam. of ☽ + angle STE,

i.e. $<$ ☽'s semi-d. + $TEC + TVC$,

$<$ ☽'s semi-d. + $TEC + ATS - TAC$,

$<$ (☽'s semi-d. + ☽'s paral.) + (☉'s semi-d. − ☉'s paral.).

A lunar eclipse will take place at, or near, opposition, if the angular distance between the centres of the moon and of the shadow is

$<$ ☽'s semi-d. + angle HTV,

$<$ ☽'s semi-d. + $THC - TVC$,

$<$ (☽'s semi-d. + ☽'s paral.) − (☉'s semi-d. − ☉'s paral.).

325. The values of the semi-diameters, parallaxes, &c., for any epoch may be calculated from solar and lunar tables,* but it is found necessary in calculating lunar eclipses to increase the diameter of the earth's shadow at the distance of the moon by about $\frac{1}{50}$th part, in order to make the result agree with observation. We may explain the

* The Nautical Almanac is always calculated for about three or four years in advance, and was commenced in 1767. When the eclipse falls in a year for which the Nautical Almanac exists, we may use the elements registered there. These are calculated for instants of time separated by regular intervals, sufficiently close to enable us to obtain the values for intermediate instants by simple rules of interpolation.

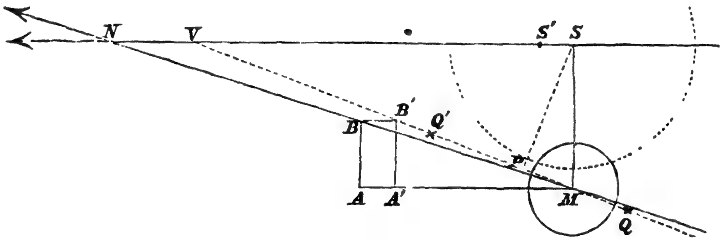
necessity for this, by remarking that HK (fig., p. 251) is determined by lines ACH, BDK which just graze the earth's surface; but such rays as AC and BD are probably absorbed by the lower strata of the earth's atmosphere, and the first rays which pass through and proceed to HK will, as it were, touch a sphere somewhat larger than the earth.

To determine the Time, Duration, and Magnitude of a Lunar Eclipse.

326. Let SN represent a portion of the ecliptic,

S being the centre of the earth's shadow
M .... moon
and SM, or λ, the latitude of the moon
at the instant of opposition,

and let the sum of the semi-diameters of ☽ and shadow $= c$.



Let MBN be the ☽'s path, and N the node,

MA, or m, the ☽'s horary motion in longitude,

AB, or p, ... latitude,

SS', or s, the $\odot$'s .... longitude.

On AM set off $AA' = SS'$, and draw $A'B'$ equal and parallel to AB, then $MB'V$ will be the *relative* path of the ☽—the earth's shadow being supposed stationary at S.

Draw SP perpendicular to MV, this will be the nearest approach of the two centres. And if, with S as centre and a radius $= c$, we describe a circle cutting MV in two points

Q, Q', these will be, on the relative orbit, the positions of the moon at the beginning and end of the eclipse; the corresponding points on the actual path may be obtained by drawing parallels through Q and Q' to MA.

327. Let $\angle AMB = \theta$ and $A'MB' = \theta'$, then θ and θ' will be known from the equations

$$\cot\theta = \frac{m}{p}, \quad \cot\theta' = \frac{m-s}{p},$$

and $$SP = MS \cos MSP = \lambda \cos\theta'.$$

If this value $\lambda \cos\theta'$ be greater than c, there will be no eclipse.

If $\lambda \cos\theta'$ be less than c, the eclipse will be partial or total:—partial, when $c - \lambda \cos\theta'$ which expresses the breadth of the portion of the moon eclipsed is less than the whole diameter; and total, when greater.

If $\lambda \cos\theta' = c$, there will be no eclipse, the disc of the moon will just graze the shadow, and the corresponding value of NS will be

$$NS = MS \cot N = \lambda \cot\theta$$
$$= c \cot\theta \sec\theta',$$

the greatest and least possible values of NS will be the superior and inferior ecliptic limits.

328. At the time t after opposition, the latitude of the ☽ will be $\lambda - pt$, and the difference of longitude of the shadow and ☽ will be $(m-s)\,t$. Therefore the distance of the centres will be

$$r = \sqrt{\{(\lambda - pt)^2 + (m-s)^2 t^2\}}.$$

The different circumstances of the eclipse may be all deduced from this formula.

If we make r equal to c, the two values of t obtained from this equation will give the times of the beginning and end of the eclipse. If the two values of t are imaginary, there will be no eclipse; if equal, the beginning and end of the eclipse

will be simultaneous, or, which is the same thing, the shadow and the moon will just come into contact, and no eclipse will take place.

If we make r equal to the difference of the semi-diameters of the moon and shadow, we shall obtain the times of the beginning and end of the *total* eclipse; but if the two values of t so obtained are imaginary, the eclipse will not be a total one.

Any value being given to r, the middle of the eclipse will correspond to the half-sum of the two corresponding times, and will be given by the formula

$$t = \frac{p\lambda}{(m-s)^2 + p^2} = \frac{\lambda}{p}\sin^2\theta'.$$

329. *Solar eclipse.* The computation of a solar eclipse, with reference to the whole earth, will be of the same character as that of a lunar eclipse, using the semi-diameter of the section EF of the cone (fig., p. 251) instead of that of HK the shadow. We can thus determine the time of beginning and ending of the eclipse generally on the earth.

But the problem becomes much more complicated when we wish to ascertain the different phases of the phenomenon, as seen at a particular place on the earth's surface. The following is a rough outline of one of the methods of proceeding:—Having found the times of the beginning and end of the general eclipse, fix upon some intermediate instant, and, for that instant, determine (by means of tables or by the Nautical Almanac) the latitudes, longitudes, parallaxes, and semi-diameters of the two bodies.

Calculate the effects of parallax on the latitudes and longitudes of each body as seen from the given place, and, by means of these reduced latitudes and longitudes, determine the apparent angular distances of the centres of the two luminaries.

This distance, compared with their apparent semi-diameters

(also corrected for parallax), will determine the magnitude of the eclipse at that instant.

We must, however, refer to other works for the detail and various modifications of the calculation (see Appendix to Nautical Almanac for 1836, by Woodhouse).

330. The occultation of a fixed star by the moon is determined by the very same methods as the solar eclipse, except that, the star having no parallax and no semi-diameter, the calculation is somewhat simplified.

331. Every phase of a lunar eclipse is visible to all parts of that hemisphere of the earth which is turned towards the moon.

When the Greenwich time of the beginning of the eclipse is known, we can find what terrestrial meridian will at that moment reckon midnight; and that place on this meridian which has its north or south latitude equal to the south or north declination of the sun will have the sun in its nadir, and therefore the moon in its zenith. If we find this place on a terrestrial globe, and describe a great circle about this place as pole, the hemisphere so marked out will see the beginning of the eclipse.

In the same manner, the hemisphere which sees the end may be determined, and the space common to the two hemispheres will see all the circumstances of the eclipse.

CHAPTER XXI.

FINDING THE LONGITUDE BY OBSERVATION.

332. THE latitude and longitude of a place on the earth's surface are the coordinates of its position. The importance of a correct determination of these elements is obvious, because, when they are known, the position of that place relatively to other known places becomes determined. The latitude is readily found by observation (Chap. IX.), but the longitude has always presented difficulties, which have only been overcome by the introduction of more delicate instruments and more refined modes of observing.

In Art. 201 it was shewn that the difference of longitude of two places is connected with the difference of the times reckoned at the two places at the same instant; and, although the problem of finding the longitude has been attempted in a great variety of ways, it will be found that they ultimately resolve themselves into finding what time is reckoned at some initial meridian (that of Greenwich for instance) at a moment corresponding to a known local time at the observer's place.*

Now, the local time is easily found by observation (Chap. XIII.); the difficulty is to obtain the corresponding time at the other meridian.

There are two ways in which this may be done:—The first consists in the observer's carrying with him the time of that other meridian, by means of a watch or chronometer,

* There is an exception in a method practised at sea and known as "dead reckoning," where the ship's place is found by noting the courses and distances run since leaving some known position. The results are, however, only rough approximations, and are neglected as soon as celestial observations can be made.

whose error and rate are known. The second method requires an observer at each station, who shall note the local time of the occurrence of some celestial phenomenon, or of a preconcerted signal which can be simultaneously seen at the two places; a comparison of the two times will determine the difference of longitude. In the case of most of the celestial phenomena, the Greenwich time of their occurring may be calculated and tabulated beforehand, so that the observer, with the Nautical Almanac in his possession, has, as it were, the simultaneous observation of the Greenwich observer.

To find the Longitude by Chronometer.

333. Let an observation be made for finding the local time by any of the methods of Chap. XIII., and let the corresponding time marked by a chronometer be noted.

Then, supposing the chronometer to have a known error and rate, that is, supposing that on some previous occasion it had been compared with Greenwich time and its error ascertained, and also its daily gain or loss, there will be no difficulty in obtaining its present error on Greenwich time, and thence the Greenwich time itself, corresponding to the local time of observation. The difference of these two times will be the longitude in time, which may be converted into degrees at the rate of 15° for every hour. The longitude will be east or west, according as the local time is greater or less than the Greenwich time.

The value of this method depends on the accuracy of the chronometer, which is *assumed* not to have changed its rate in the interval. As the best instruments are liable to irregularities, the danger is lessened by the employment of several chronometers, and the longitude is then deduced by taking a mean of the results;—more or less weight being given to the indication of each instrument according to our previous experience of its accuracy. The method by chronometers is especially valuable at sea.

By Electric Telegraph.

334. The difference of longitude between two places P and Q may be very accurately determined when these places are connected by an electric telegraph.

Let a signal be made at P, the more easterly station, at the time T_1 of P; and suppose the signal to be received at Q at the time T_2 of Q (T_1 and T_2 being both solar, or both sidereal, times obtained from the clock times at the two stations by correcting for the errors of the clocks).

If the transmission of the electric current were instantaneous, the difference of longitude would be

$$\lambda_1 = T_1 - T_2,$$

but if x be the time required for the transmission of the signal, $T_1 + x$ will be the time at P corresponding to the time T_2 at Q, therefore the difference of longitude is

$$\lambda = (T_1 + x) - T_2 = \lambda_1 + x.$$

To eliminate the unknown quantity x:—Let a signal be made at Q at the time T_2', and received at P at the time T_1'; then

$$\lambda_2 = T_1' - T_2'$$

would be the difference of longitude if no time were lost in the transmission. Assuming this time to be again x,

$$\lambda = T_1' - (T_2' + x) = \lambda_2 - x,$$

whence $$\lambda = \tfrac{1}{2}(\lambda_1 + \lambda_2) \text{ is known.}$$

335. In the practice of this simple method, when we wish to obtain from it the very great accuracy of which it is capable, we have to take into account certain small possible errors. Such are

The errors of the assumed clock corrections,

The personal errors of the observer who gives, and of him who receives, the signal (Art. 83),

The error due to neglecting the small fraction of time

required to complete the galvanic circuit after the finger touches the signal key,

&c. &c.

For a full examination of these errors, and of the means of eliminating them, or of reducing them to a minimum, we shall refer to Chauvenet's *Astronomy*, vol. I., where the student will also find a description of the method called ' by star signals," which is a slightly modified way of employing the electric telegraph.

336. For places which are not connected by electric communication, the difference of longitude may be obtained, when the stations are not too far apart, by observing simultaneously the time marked by the clock at each station when some signal, such as a flash of gunpowder, or the disappearance or reappearance of some fixed light, is made either at one of the stations or at some intermediate point. The method may be extended to find the difference of longitude of distant places when they can be linked by intermediate stations.

By Celestial Signals.

337. Some celestial phenomena are of such a character that they may be observed at the same instant of absolute time by different observers. Such are—

1. The eclipses of Jupiter's satellites.
2. The beginning or end of a lunar eclipse.
3. The bursting of a meteor.

Any one of these being observed at two stations, the difference of the corresponding local times will give the difference of longitude.

The exact instants of the disappearance of Jupiter's satellites into the shadow of the planet, and of their reappearance, are predicted in the Nautical Almanac for Greenwich time; so that an observer noting *his* local time of either phenomenon, has at once a means of inferring his

absolute longitude. But the method is not susceptible of very great accuracy; for, the disappearance of a satellite taking place gradually, the apparent instant of disappearance, and therefore the longitude inferred, will depend on the power of the telescope employed; and, at sea, the motion of the ship renders it impossible to keep the planet in the field of view of a telescope of sufficient power to shew the satellites.

The method by lunar eclipses is still more uncertain; for, the boundary of the earth's shadow on the moon is so indefinite, that observers, at the same place, will differ by two or three minutes in their estimation of the time of beginning or ending of the eclipse.

The bursting of meteors, or shooting stars, might be useful if it were possible to anticipate them, and also to identify them when observed.

By Lunar Distances.

338. The moon has a rapid motion among the stars, altering its position by more than 13° in 24 hours. Certain bright stars, and some of the planets, are selected, which lie almost directly in the moon's path, and the angular distances of the moon's centre from these and also from the sun (as they would appear to an observer at the centre of the earth), are given in the Nautical Almanac for every third hour of Greenwich mean time.

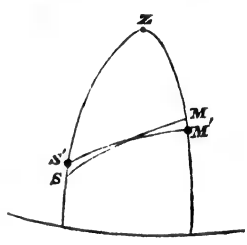
It will be easy to understand how this is made available for finding the longitude, whether at sea or on shore:—Measure, with a sextant, the angular distance between the moon's bright limb and a star, and correct this *observed distance* for instrumental errors. Then, add or subtract the moon's apparent semi-diameter according as the bright limb is towards or from the star; this will give the *apparent distance* of the centres.

When the sun is the body whose distance from the moon is measured, the apparent semi-diameters of both bodies are always to be added, because the bright limb of the moon is turned towards the sun, and the observed distance is that of the nearest limbs.

The apparent distance so found must be further corrected for the effect of parallax and refraction, as explained below, to determine the geocentric or *true distance.* This true distance, compared with the distances registered in the Nautical Almanac, will determine the time at Greenwich, and thence, by comparison with the local time of observation, the longitude of the place.

339. Only one observation has been supposed made, that of the distance; but we want the local time to compare with the Greenwich time, therefore an altitude also must be simultaneously observed (Chap. VIII.). Moreover, the operation of *clearing the distance*, as the computation of the true distance is called, requires both the altitudes to be known. There should therefore be three observers, one measuring the distance, and the others, at the same moment, the altitudes of the bodies.* The local time may be obtained from either altitude.

340. Let Z be the zenith of the observer, S' the apparent place of the star or sun, and M' that of the moon. We shall assume that the effects of parallax and refraction are wholly in the vertical planes ZS' and ZM',† and that the true places are S and M. The parallax of the moon exceeds its refraction, therefore M' is



* When there is but one observer he may proceed as follows:—Take the altitude of the sun or star, then the altitude of the moon, next the distance between the luminaries, again the moon's altitude, and finally that of the sun or star. The intervals of time having been also noted by a watch, the observed changes of altitude may, for a short time, be assumed to proceed uniformly, and will enable him to calculate what the altitudes would have been at the moment when the distance was taken.

† Strictly, parallax acts in a plane through the geocentric zenith, and refraction in a plane through the astronomical zenith, but the error introduced is very much less than the probable error of observation, and may therefore be safely neglected.

below M, and the contrary for the sun, a planet, or a star, therefore S' is above S.

The apparent zenith distances ZS', ZM' are observed at the same time as the distance $S'M'$. From these the true zenith distances ZS, ZM must be found by correcting for parallax and refraction.

In the triangle $ZS'M'$, the three sides being known, the angle Z may be calculated.

Then, in the triangle ZSM, the two sides ZS, ZM and the contained angle Z will be known, and the true distance SM may be obtained.

341. The solution of these two triangles is a simple application of the rules of Spherical Trigonometry.

Let m', s' be the apparent altitudes of the moon and star,

m, s ...... true ....,

d' ...... apparent distance of their centres,

d ...... true ....

If the angle MZS be called ϕ, we have

$$\cos d' = \sin m' \sin s' + \cos m' \cos s' \cos\phi,$$

$$\cos d = \sin m \sin s + \cos m \cos s \cos\phi,$$

whence $$\frac{\cos d - \sin m \sin s}{\cos m \cos s} = \frac{\cos d' - \sin m' \sin s'}{\cos m' \cos s'} \;......(A).$$

This determines d, but the equation is not in a form adapted to logarithmic computation. To prepare it for this purpose, add 1 to each side,

$$\frac{\cos d + \cos(m+s)}{\cos m \cos s} = \frac{\cos d' + \cos(m'+s')}{\cos m \cos s},$$

$$\frac{(1 - 2\sin^2 \tfrac{1}{2}d) + \{2\cos^2 \tfrac{1}{2}(m+s) - 1\}}{\cos m \cos s}$$

$$= \frac{2\cos\tfrac{1}{2}(m'+s'+d')\cos\tfrac{1}{2}(m'+s'-d')}{\cos m' \cos s'},$$

$$\sin^2 \tfrac{1}{2}d = \cos^2 \tfrac{1}{2}(m+s) - \frac{\cos m \cos s}{\cos m' \cos s'} \cos\tfrac{1}{2}(m'+s'+d')\cos\tfrac{1}{2}(m'+s'-d').$$

$$\text{Let } \sin^2\theta = \frac{\cos m \cos s}{\cos m' \cos s'} \frac{\cos \frac{1}{2}(m'+s'+d') \cos \frac{1}{2}(m'+s'-d')}{\cos^2 \frac{1}{2}(m+s)} \quad \ldots\ldots\ldots\ldots\ldots (B);$$

therefore
$$\sin^2 \tfrac{1}{2}d = \cos^2 \tfrac{1}{2}(m+s)(1-\sin^2\theta),$$
$$\sin \tfrac{1}{2}d = \cos \tfrac{1}{2}(m+s) \cos\theta \quad \ldots\ldots\ldots\ldots\ldots (C).$$

The formulæ (B) and (C) determine d, and are adapted to logarithms. The above is Borda's solution, which is perfectly general and requires no distinction of cases.

342. We shall give another solution which has found great favour with mariners on account of its simplicity. It requires a table of natural versed sines.

Subtract 1 from each side of equation (A),

therefore
$$\frac{\cos d - \cos(m \sim s)}{\cos m \cos s} = \frac{\cos d' - \cos(m' \sim s')}{\cos m' \cos s'}.$$

Assume
$$\frac{\cos m \cos s}{\cos m' \cos s'} = 2\cos\theta \quad \ldots\ldots\ldots\ldots\ldots (D);$$

therefore
$$\begin{aligned} \cos d &= \cos(m \sim s) + 2\cos\theta\,\{\cos d' - \cos(m' \sim s')\} \\ &= \cos(m \sim s) + \cos(d'+\theta) \\ &\quad + \cos(d'-\theta) - \cos\{(m' \sim s')+\theta\} - \cos\{(m' \sim s')-\theta\}, \end{aligned}$$

whence,
$$\begin{aligned} \operatorname{vers} d &= \operatorname{vers}(m-s) + \operatorname{vers}(d'+\theta) \\ &\quad + \operatorname{vers}(d'-\theta) - \operatorname{vers}(m'-s'+\theta) - \operatorname{vers}(m'-s'-\theta) \ldots (E). \end{aligned}$$

343. Many other transformations have been proposed, but the above are among the simplest. Instead of obtaining the true distance d directly, which, on account of its being a large angle, will require extended tables and great care in working for the proportional parts, we may employ approximative methods which find the difference between d and d'. As this difference will always be a small angle seldom exceeding 1°, the work will be less troublesome, without being practically less correct. The value of $d - d'$ will be found by successive approximation, or by development in a series in which the smaller terms are neglected.

344. The true distance having been found, we have to compute the corresponding Greenwich time. Referring to the Nautical Almanac, we find the calculated distance comprised between some two of those registered there; and, on the assumption of a uniform variation of the distance during those three hours, a simple proportion will give the correction to be added to the first hour.

If d be the true distance; d_1, d_2 the distances, given in the tables, separated by three hours; x the correction in hours,

$$d_2 - d_1 : d - d_1 :: 3 : x,$$

$$\log x = \log 3 + \log (d - d_1) - \log (d_2 - d_1).$$

345. *Proportional logarithms.* This calculation is very much simplified by the use of a special table called table of proportional logarithms, the idea of which is due to Dr. Maskelyne, the Astronomer Royal, at whose suggestion the yearly publication of the Nautical Almanac began in 1767. The above proportion may be written

$$\frac{3}{x} = \frac{d_2 - d_1}{d - d_1} = \frac{3}{d - d_1} \div \frac{3}{d_2 - d_1}.$$

Now the table of prop. logs. is so constructed that opposite to any number a stands $\log \frac{3}{a}$, therefore

$$\text{prop. } \log x = \text{prop. } \log (d - d_1) - \text{prop. } \log (d_2 - d_1).$$

The Nautical Almanac gives not only d_1 and d_2 but also in a column between them prop. $\log (d_2 - d_1)$; therefore at the same time that we take out d_1, the distance which precedes the true distance d, we take out prop. $\log (d_2 - d_1)$, and we only have two inspections of the logarithmic tables instead of four.

346. We have supposed the distance between the moon and a star to increase or decrease uniformly; but this will

not generally be the case, and the result obtained has to be further corrected by the usual rules of interpolation. The Nautical Almanac contains a table which gives the necessary correction for second differences in a convenient form. See Boole's *Finite Differences*, Chap. III., on 'Interpolation.'

347. The horizon at night is frequently so ill-defined that there is a large probable error in the altitude of the star, and the local time determined from it will have a corresponding uncertainty, although the altitude would be sufficiently accurate for clearing the distance. But as all vessels engaged in distant voyages are furnished with one or more chronometers, we may employ the observed lunar distance to determine—not the longitude of the ship—but the error of the chronometer on Greenwich time at the instant of observation; then, at any convenient time afterwards, determine the longitude by an observation of the sun and chronometer.

We may even dispense with the observation of the altitudes altogether at the time of taking the distance; because, the supposed Greenwich time, given by the chronometer and the estimated longitude, will be sufficiently accurate to determine the time at ship, and thence the hour angle, which, with the known latitude and the declinations of the star and moon, will lead to the altitudes with sufficient accuracy for clearing the distance.

The reader who is interested in the problem may consult Delambre's *Astronomie*, Chap. XXXVI.; Mackay on *Longitude*, Bk. 3, Chap. IV.; Chauvenet's *Astronomy*.

The method of lunar distances for determining the longitude owes its value to the rapid motion of the moon among the other heavenly bodies. If it moved twice as fast, the accuracy would be twice as great. The inhabitants of Jupiter must therefore, with their rapidly revolving moons,

be able to determine their longitudes with as great accuracy as we can our latitudes.

By Moon-Culminating-Stars.

348. The preceding method for the determination of the longitude depends for its success on the rapid change of the moon's distance from certain fixed stars. The change in its right ascension, which is equally rapid and sometimes even more so, is the basis of another method, not available at sea, but which, on land, leads to very accurate determinations.

A transit instrument being mounted in the meridian plane, the instant is noted by an astronomical clock when the bright limb of the moon comes to the meridian, and also the time of transit of a selected star, taken from the list of moon-culminating-stars in the Nautical Almanac. These moon-culminating-stars are chosen, for each day, so as to have nearly the same declination as the moon, and not to differ very widely from it in right ascension.

The difference between the two times so observed is the difference of the right ascensions of the star and of the moon's bright limb, at the moment when the latter is in the meridian of the observer.

If similar observations be made at Greenwich, we shall have the difference of right ascensions of the star and of the moon's limb when the latter is in the meridian of Greenwich.

We can thus find the *change* in the right ascension of the moon's limb while it passes from the one meridian to the other; and if this change be divided by the variation corresponding to *one* hour of longitude,* which is nearly uniform,

* The column is headed 'Variation of ☾'s right ascension in one hour of longitude,' but it is calculated for the bright limb, and therefore includes the effect of a change of the semi-diameter. (See Nautical Almanac Explanation.)

and is given in the Nautical Almanac for the instant of the passage over Greenwich, we shall obtain the difference of longitude in time.

349. Instead of corresponding observations at Greenwich, we may take the right ascensions of the moon's limb and of the star as registered in the Nautical Almanac, the computed places being now given with such accuracy that they may be considered the same as if they had been observed.

When the two places differ considerably in longitude, we must use for our divisor, not the variation for one hour given in the tables, but one obtained by interpolation corresponding to the middle of the interval between the two observations.

Let T_1, T_2 be the observed clock times of transit of ☽'s limb and star,

A_1, A_2 be the registered right ascensions of same at Greenwich transit,

C be the registered change in one hour of longitude corresponding to the middle time.

$$\text{Then, longitude} = \frac{(T_1 - T_2) - (A_1 - A_2)}{C}.$$

Both numerator and denominator must be expressed in seconds, and the result will be the longitude expressed in hours and fractions of an hour—west, if positive; east, if negative.

350. The advantage of selecting stars which have nearly the same declination as the moon is, that the instrumental errors will affect both the bodies equally. The longitude so determined must yet be corrected for the known errors of the transit (Chap. IV. Art. 82); for, if we suppose the star to be observed t^s too soon, we are in reality observing the transits across the meridian of an observer t^s to the east of ours, and the longitude obtained above will be his longitude, which must therefore be increased by t^s.

351. The longitude of a place may also be found by an eclipse of the sun, or by an occultation of a star or planet, but the operation is much too intricate for an elementary work, and the rare occurrence of these phenomena renders the methods of less practical value than those we have here considered. For a complete investigation of these, and of some other methods of determining the longitude of a place, we shall refer to Chauvenet's *Astronomy.*

CHAPTER XXII.

THE PLANETS.

352. We have already, on several occasions, spoken of the planets—those wandering stars whose brilliancy and motions among the other stars must from the very earliest times have attracted attention. Five planets were known to the ancients:—*Mercury*, *Venus*, *Mars*, *Jupiter*, and *Saturn*; and the motions of these five were all found confined within a narrow zone of less than 8° on each side of the ecliptic. To this zone they gave the name of *zodiac*.

No additions were made to the planetary system before the end of the 18th century. On the 13th of March, 1781, Sir W. Herschel discovered *Uranus*, "in the course of a review of the heavens, in which every star visible in a telescope of a certain power was brought under close examination, when the new planet was immediately detected by its disc, under a high magnifying power."

On the 1st day of January, 1801, another planet *Ceres* was added. This was the first of a series of very small telescopic objects, the number of which, now exceeding a hundred, seems likely to go on increasing with the increase of magnifying power of our telescopes. They are not, like the other planets, confined to the zodiac, and it has been conjectured that they are probably fragments of some larger planet, blown to pieces by an explosion.

A still more remarkable planetary discovery was made some years ago almost simultaneously by Professor Adams

and Mons. Le Verrier. The history of this belongs to physical astronomy, and we shall refer the reader to the full account given in Grant's valuable work;* a few words, however, will explain the peculiar circumstances which led to the discovery, and which mark it as one of the most wonderful in the whole range of astronomy. The planet Uranus discovered, as we have said above, by Herschel, was carefully observed for a series of years; and, these observations serving for basis, the subsequent path of the planet was calculated and registered, taking carefully into account all *known* causes of disturbance. In a few years it was found that the positions of the planet did not exactly agree with the calculated positions, and it was surmised that the *unknown* cause of disturbance was probably an undiscovered planet. Adams and Le Verrier proposed to themselves, and successfully achieved, the solution of the inverse problem: "Given the unaccounted-for disturbances of the planet Uranus, to find the position, &c., of the unknown disturbing body." The indications furnished by their calculations led to the discovery of the planet Neptune.

353. When a planet is attentively observed, and its position marked down on the celestial sphere or map, night after night, it is soon found that its path differs essentially in character from those of the sun and moon. These luminaries have a motion which, though not strictly uniform, departs but little from uniformity; but, in the case of the planets, we find, not only a want of uniformity, but an actual change of direction. The planet, after advancing for some time from west to east in the same direction as the sun and moon, gradually relaxes its speed, then stops, and begins to retrograde. After retrograding, with a velocity at first increasing and then diminishing, it again stops, and then recommences

* *History of Physical Astronomy.*

its onward motion. The time of regression is, however, much less than that of progression, and there is on the whole an advance from west to east.

The ancients, who started with the fixed idea of an immoveable earth, found these retrograde motions and stationary positions a source of much perplexity, and, to explain them, were driven to complicated systems of epicycles. The planet was supposed to describe a circle uniformly, the centre of this circle itself moving on another circle, and so on. But, however ingenious these systems were, they could not completely reconcile the observations.

354. We shall not enter into a description of the various hypotheses which preceded that of Copernicus (1543). This celebrated astronomer shewed that all these different and complicated planetary phenomena could be easily and satisfactorily explained by supposing the earth to be itself a planet, circulating round the sun in the same direction as the other planets. According to this system, which has long ceased to meet with the antagonism which its supposed contradiction of experience and revelation originally excited, the sun occupies the centre, and the planets succeed one another in the following order, reckoning from the sun outwards. The mean distance of the earth is taken for unit, and the period is expressed in days.

	Mean Distance from Sun.	Period.
Mercury	0·3871	87·969
Venus	0·7233	224·700
Earth	1·0000	365·256
Mars	1·5237	686·980
Minor Planets (average)	2·6	1531·
Jupiter	5·2028	4332·585
Saturn	9·5389	10759·220
Uranus	19·1827	30686·821
Neptune	30·0370	60126·720

The two planets, Mercury and Venus, whose orbits are within that of the earth, are called *inferior* planets, all the others are *superior* planets.*

355. By a careful and elaborate study of the motions of the planet Mars, Kepler was led to the discovery of his three laws (Chap. XI.), which proved that the idea of circular motion, with its attendant epicycles, must be discarded for the much simpler one of motion in a conic section round the sun in the focus; and Newton confirmed Kepler's discoveries by shewing them to be necessary consequences of his law of universal gravitation.

In order, then, to see the motions of the planets in their greatest simplicity, we must conceive the observer stationed at the sun. But it is obvious that the positions which the planets would then occupy, relatively to the stars, would be very different from those which they seem to have when seen from the earth. The two positions are known as the *heliocentric* and the *geocentric* places of the planet; and, when the dimensions and positions of the elliptic orbits both of the earth and of the planet are known, as also the places the bodies occupy in them at any assigned time, it will be a simple application of the rules of plane trigonometry to convert heliocentric into geocentric latitudes and longitudes, and *vice versâ*.

* The distances of the principal planets from the sun may be roughly obtained by a somewhat curious law, known as *Bode's law*, given by him in 1778.

Write down the series of numbers, 0, 3, 6, 12, 24, 48, 96, 192, 384; where, after the second, each is formed by doubling the preceding; next, add 4 to each and divide by 10. We get values approximately coinciding with the planetary distances, viz.—

Mer.	Venus	Earth	Mars	Mi. Pls.	Jupiter	Saturn	Uranus	Neptune
·4	·7	1·0	1·6	2·8	5·2	10·0	19·6	38·8

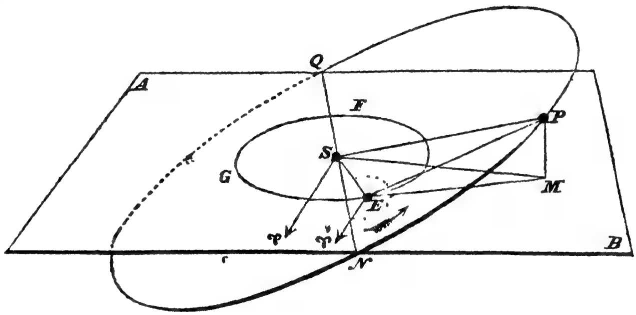
When Bode's law was first given, there was a gap between Mars and Jupiter, which the minor planets subsequently occupied. The discovery of Neptune, however, disturbs the series too much to allow us to attach any great value to the law, beyond that of being a simple means of remembering the distances.

Thus, suppose S to represent the sun, AB the plane of the ecliptic, EFG the earth's orbit, and E the earth's place; NPQ a planet's orbit and P the place of the planet at the same time; $S\Upsilon$ and $E\Upsilon'$ parallel lines in the direction of the first point of Aries. Then, drawing PM perpendicular to the plane of the ecliptic,

PSM and ΥSM are the heliocentric lat. and long. of P,

PEM and $\Upsilon' EM$ ....geocentric ....

The heliocentric longitude of the earth (ΥSE) differs by 180° from the geocentric longitude of the sun ($\Upsilon' ES$, measured in direction of the arrow).



The heliocentric distance SP and the latitude PSM being known, the right-angled triangle PSM will furnish PM and the *curtate* distance SM.

SE and SM being known, and the angle ESM (the difference of heliocentric longitudes of planet and earth), EM and the angle SEM may be obtained, and thence the angle $\Upsilon' EM$, which is the geocentric longitude.

The geocentric latitude will also be readily obtained from the right-angled triangle PEM, of which PM and EM have been calculated.

In a somewhat similar manner may a geocentric position be converted into a heliocentric one.

356. The heliocentric place of a planet can be calculated by the rules of elliptic motion, when certain quantities are

known which determine the dimensions of the elliptic orbit; its position with reference to the ecliptic, and the position of the planet itself in the orbit at some known instant.

These necessary data are called the *elements of the planet*, and are six in number—

1. The semi-axis of the ellipse,
2. The excentricity,
3. The heliocentric longitude of the node,
4. The inclination,
5. The longitude of the apse,
6. The epoch.

The line of nodes is the intersection NQ of the plane of the orbit and the ecliptic, the point N where the planet passes from the south to the north side of the ecliptic, being the ascending node (☊), the other Q the descending node (☋). The heliocentric longitude of the node is always the longitude (♈SN) of the ascending node, therefore (3) and (4) fix the position of the plane of the orbit.

The position of the apse line, given by (5), indicates the direction of the major axis; the magnitude of the ellipse is completely determined by (1) and (2). The epoch (6) is the longitude of the planet at a certain definite instant, and this being known, we may, by the rules of elliptic motion (Chap. XI., Art. 184), obtain the position of the planet in the orbit at any subsequent instant.

357. Before the place of a planet can be predicted, it is therefore necessary to determine its elements. Three complete observations of right ascension and declination will be sufficient; for from each observation can be inferred the corresponding geocentric latitude and longitude, and these will furnish two equations connecting the elements of the orbit. But although theoretically sufficient, it will be found practically more accurate and more simple to make observations at particular times, when the planet occupies selected positions

specially favourable for finding each element in turn. We shall refer the reader to Delambre's *Astronomy*.

358. *To find the planet's periodic time.* Let T be one year, and t the periodic time of a planet. Let the synodical period S be observed, that is, the interval of time from conjunction to conjunction, or from opposition to opposition, or, generally, from any one position relatively to the sun to the same position again. Then $\frac{2\pi}{T}$ and $\frac{2\pi}{t}$ are the true angular velocities of the earth and planet, and $\frac{2\pi}{S}$ their relative angular velocity, therefore

$$\frac{2\pi}{T} \sim \frac{2\pi}{t} = \frac{2\pi}{S};$$

therefore $$\frac{1}{t} = \frac{1}{T} - \frac{1}{S} \text{ for a superior planet,}$$

$$\frac{1}{t} = \frac{1}{T} + \frac{1}{S} \text{ for an inferior planet,}$$

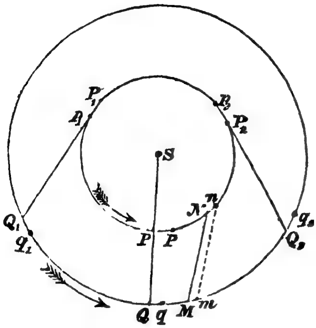
whence, the periodic time of a superior planet is $\frac{ST}{S-T}$, and of an inferior one $\frac{ST}{S+T}$.

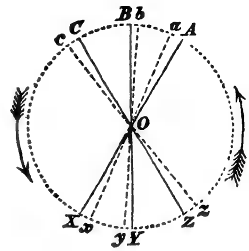
Stationary Points.

359. We shall proceed to shew how the stationary positions and the retrograde motions of planets are necessary consequences of the earth's own motion.

If we consider elliptic orbits, the problem of the determination of the stationary points becomes a very complicated one; but, as these points are now objects of interest only from the fact of the ancients having been so puzzled to explain them, it will be sufficient to investigate their positions, considering the orbits as circles uniformly described round the sun as a centre.

Let S be the sun, P and Q two planets in conjunction, the direction of motion being indicated by the arrows.





By Kepler's third law,

$$\text{period of } P : \text{period of } Q :: (SP)^{\frac{3}{2}} : (SQ)^{\frac{3}{2}},$$

and if Pp, Qq be small arcs described in the same time,

$$Pp : Qq :: \text{vel. of } P : \text{vel. of } Q$$

$$:: SP \times \text{ang. vel. of } P : SQ \times \text{ang. vel. of } Q$$

$$:: \frac{SP}{\text{periodic time of } P} : \frac{SQ}{\text{periodic time of } Q}$$

$$:: \frac{1}{SP^{\frac{1}{2}}} : \frac{1}{SQ^{\frac{1}{2}}};$$

therefore, $Pp > Qq$.

Let P_1, Q_1 be positions of P and Q before conjunction, at the moment when the line joining them is a tangent to the inner orbit, and let P_2, Q_2 be the corresponding positions after conjunction.

Firstly, take the case of an *inferior* planet. Let P represent it, and let Q be the earth.

While the planet moves from P_1 to p_1, let the earth move from Q_1 to q_1. Take a fixed point O to represent the supposed fixed position of the observer, and draw OA parallel to Q_1P_1, Oa parallel to q_1p_1. The apparent direction of the planet's motion will therefore be Aa, in the direction of the arrow, *i.e.* direct. When the planet arrives at P and the earth at Q,

draw OB to represent the apparent direction; then, since $Pp > Qq$, the direction will change to Ob parallel to qp, and the apparent motion is retrograde.

Again, in the position Q_2P_2 to which OC is parallel, the motion of the planet alone, from P_2 to p_2, would cause no change in the direction, but the earth moving to q_2 at the same time determines an apparent direction Oc parallel to q_2p_2, and the motion is again direct.

While the earth, then, is passing from Q to Q_2, the apparent motion of the planet changes from retrograde to direct; at some intermediate point, therefore, as M, the retrograde motion ceases in order to become direct, and the line MN, joining the two bodies, moves parallel to itself,—the greater velocity of N being compensated by its moving more obliquely. The planet is then stationary. In the same way, it may be shewn, that there must be a stationary point before conjunction, between Q_1 and Q.

Secondly, consider a *superior* planet, and take P (same figure) now to represent the earth, and Q the superior planet.

When S, P, Q are, as in the figure, in one straight line, the planet Q is in opposition. Make the same construction as before, and produce the lines AO, BO, CO to X, Y, Z; then OX, OY, OZ are apparent directions of the planet, and a simple inspection of the figure will shew that at Q the apparent direction of motion will be retrograde, but direct at Q_1 and Q_2; that is, the motion of each planet, as seen from the other, will be exactly the same at the same time—both retrograde, both direct, or both stationary.

360. We may determine the stationary position analytically as follows:

Let t be the time from the positions Q, P to the stationary points M, N. Join SM and SN (these lines are not drawn in the figure). Let $SM=a$, $SN=b$, and let ϕ, ϕ' represent

the angles M and N of the triangle SMN. Also let α, β be the angular velocities of the bodies

$$QSM = \alpha t, \quad PSN = \beta t, \quad MSN = (\beta - \alpha)\, t.$$

In a small time τ, MN moves to mn parallel to itself,

$$Mm \sin mMN = Nn \sin nNM,$$

or,
$$a\alpha\tau \cos\phi = -b\beta\tau \cos\phi'.$$

By Kepler's law $\quad \alpha : \beta :: b^{\frac{3}{2}} : a^{\frac{3}{2}}$;

therefore
$$b^{\frac{1}{2}} \cos\phi = -a^{\frac{1}{2}} \cos\phi'.$$

Also
$$a \sin\phi = b \sin\phi',$$

$$\cos\phi = \sqrt{\left(\frac{a^2 + ab}{a^2 + ab + b^2}\right)}, \quad \sin\phi = \sqrt{\left(\frac{b^2}{a^2 + ab + b^2}\right)},$$

$$\cot\phi = \sqrt{\left\{\left(\frac{a}{b}\right)^2 + \frac{a}{b}\right\}} \text{.........(i)},$$

$$\cos\phi' = \sqrt{\left(\frac{ab + b^2}{a^2 + ab + b^2}\right)}, \quad \sin\phi' = \sqrt{\left(\frac{a^2}{a^2 + ab + b^2}\right)},$$

$$\cot\phi' = \sqrt{\left\{\left(\frac{b}{a}\right)^2 + \frac{b}{a}\right\}} \text{.........(ii)},$$

$$\cos(\beta - \alpha)\, t = -\cos(\phi + \phi') = \frac{\sqrt{(ab)}\,\{a + \sqrt{(ab)} + b\}}{a^2 + ab + b^2}$$

$$= \frac{\sqrt{(ab)}}{a + b - \sqrt{(ab)}} \text{...............(iii)};$$

(i) and (ii) determine the elongation of each planet as seen from the other at the moment when they appear stationary, and (iii) gives the difference of their heliocentric longitudes and the time from conjunction. The angles QSM and PSN can of course be found when t is known.

Transit of Venus.

361. Since the orbit of the planet Venus is within that of the earth, it will sometimes happen that the planet will find itself immediately between the sun and the earth, and a phenomenon analogous to an annular eclipse of the sun will

take place, except that the planet will hide only a very small part of the sun, and will appear like a dark spot moving across the sun's disc. The motion also, contrary to that of the moon in a solar eclipse, will be from east to west, because the planet is then retrograding. This, which is called a transit of Venus, furnishes the most accurate method for the determination of the sun's parallax.

The phenomenon, being caused by the interposition between us and the sun of a nearer opaque body, will, as in the case of a solar eclipse, present local features which will vary from one observer to another. Neither the beginning nor the end of the transit will happen at the same absolute instant of time for all observers, nor will the chord of the sun's disc, which Venus seems to describe, be the same for all. These variations are due to the different distances of the two bodies. Venus being then at only about $\frac{2}{7}$ths of the sun's distance, a displacement of the observer will cause a much greater apparent displacement of the planet than of the sun.

The non-simultaneous occurrence is the basis of Delisle's method of determining the parallax; the non-coincident path is the basis of Halley's. We shall give a slight outline of the principles of these two methods. The necessary calculations for strict investigations are long, being of the same character as those for a solar eclipse, and we shall refer the reader to Chauvenet's *Astronomy* or to Delambre's *Astronomy*, Vol. II.*

362. *Delisle's Method.* Consider a cone enveloping both the sun and the earth with its vertex beyond the earth. That generating line of the cone which is first touched

* Mr. Proctor's valuable papers on the transit of Venus are in the highest degree suggestive, and shew not only a complete mastery of the problem, but a power of putting it in a clear and simple light, which but few writers possess. Mr. Proctor's writings on astronomical subjects deserve, and will well repay, careful study.

by Venus will determine on the earth the place of first contact. Consider a second cone enveloping the sun, and with its vertex at the centre of the earth; the transit will begin for the centre of the earth when Venus first touches this cone. Lastly, consider a double cone enveloping the earth and sun, and having its vertex between them (the distance of this vertex from the earth will be only $\frac{1}{109}$ of the sun's distance); the first contact of Venus with this cone will determine that place on the earth's surface at which the beginning of the transit is most retarded.

For intermediate stations the transit will begin at intermediate times, the ingress being accelerated for some and retarded for others. The interval between the earliest ingress and the latest will, when the transit is a central one, amount to about 12 minutes; in other cases it will be longer—in 1874 it will be 25 minutes. During this interval the earth will have turned on its axis through only a small angle, and the places of most accelerated and of most retarded ingress will be nearly the antipodes one of the other. At both places the sun will be just in the horizon.

If now, at any two stations situated as near these points as will be most practicable and convenient, the instant of ingress* be observed, and *if the longitudes of these stations are accurately known* then the difference of absolute times will be known. Since this difference is entirely owing to the excess of the parallax of Venus over that of the sun, it will be possible to calculate this excess, and the ratio of the parallaxes is known from the known ratio of the distances, therefore the parallaxes themselves can be found.

The parallaxes will also be similarly determined by

* The contact here spoken of must be the internal contact of Venus and the sun; the external contact of ingress could scarcely be determined with accuracy, since Venus would be perceived only *after* having encroached on the sun's disc.

observations on the egress of the planet at places near where the egress is most accelerated and where it is most retarded.

363. *Halley's Method.* Instead of considering the absolute interval between an accelerated and a retarded ingress, let us take two stations in opposite hemispheres, at each of which both the ingress and the egress can be seen, but such that the chords of the sun's disc which the planet seems to describe may be widely separated.

These chords having different lengths will be described in different times; and if the instants of ingress and of egress be observed at each station, the difference of duration of the two transits will be known. This difference is again owing to the difference of the parallaxes, and will furnish an equation for determining it, which, with their known ratio, will, as in the former method, lead to the parallaxes themselves.

364. The advantage of Halley's method consists in its requiring only the *duration* of the transit to be accurately measured, and an ordinary chronometer whose rate is known will do this; so that no lengthened stay on what would probably prove an uninhabitable land is necessary, nor any delicate observation for the determination of the longitude.

The advantage of Delisle's method consists in its requiring only the beginning *or* the end of the transit to be observed; so that many more stations will be suitable to the application of the method, and the difficulty of ascertaining the longitude of each station with the necessary accuracy will probably be more than compensated by the multiplied observations which can be made.

365. The following additional remarks will explain a little more fully how the sun's parallax is deduced from the observations. We shall consider Delisle's method:

Let T be the Greenwich time, approximately known, at which the first internal contact takes place for an observer at the centre of the earth. Suppose an accelerated ingress to have been observed at the time $T-t$ at some place whose position is accurately known. Let a, b be the differences of right ascension and of declination of Venus and the sun at the time T, and let α, β be the relative horary changes, so that $a+\alpha t$, $b+\beta t$ are the values at the instant of observation.

Let ϖ be the sun's parallax, $k\varpi$ that of Venus; k being the known ratio of the distances at the moment of observation. The effects of parallax on the right ascension and declination (δ) of the sun for the given place at the instant of observation will be ϖX and ϖY, where X and Y are certain functions of the hour angle, declination and latitude. Since the positions are nearly coincident, the corresponding corrections for Venus will be $k\varpi X$ and $k\varpi Y$, and the *apparent* differences of right ascension and of declination at the moment of observation are

$$a+\alpha t-(k-1)\varpi X, \quad b+\beta t-(k-1)\varpi Y.$$

If c be the difference of semi-diameters of Venus and the sun

$$a^2\cos^2\delta+b^2=c^2,$$

$$\{a+\alpha t-(k-1)\varpi X\}^2\cos^2\delta+\{b+\beta t-(k-1)\varpi Y\}^2=c^2;$$

whence, subtracting, and neglecting squares and products of the small quantities αt, βt, and ϖ,

$$t=\frac{aX\cos^2\delta+bY}{a\alpha\cos^2\delta+b\beta}(k-1)\varpi=M\varpi,$$

so $\quad t'= \quad M'\varpi,$

M' being the value of M at the second station, where a retarded ingress is observed at the time $T+t'$; then

$$\varpi=\frac{t+t'}{M+M'}, \text{ will be known,}$$

because $t+t'$, the absolute time interval, is known from the

observations, when the longitudes of the two places are accurately known.

366. There will also be transits of the planet Mercury, and they will recur much more frequently than those of Venus; but, on account of Mercury's proximity to the sun, the intervals between the accelerated and the retarded ingress or egress will be much less, and the durations of transits be much more nearly equal than in the case of Venus; and as it is especially on the magnitudes of the intervals and on the differences of the durations that the accuracy of the methods depends, the transits of Mercury are comparatively valueless for the determination of the sun's parallax.

367. The transits of Venus take place at irregular and distant intervals. The last occurred in 1769, and only three have yet been seen, viz. in 1639, in 1761, and 1769. That of 1769 was observed from a great number of different stations on the surface of the earth, and the results of the observations gave the parallax of the sun 8″·57, which, until recently, had been adopted as a very approximate value; but there is reason to suppose that this number is too small; and independent observations, of a totally different kind, made with all the refinement of which modern astronomy is capable, have led to the adoption of 8″·93 (Chap. XVI. Art. 261).

A transit does not recur at every conjunction, for the same reason that an eclipse of the sun does not take place at every new moon. The inclination of the orbit gives to the planet a latitude which places it beyond the sun's disc, and thereby avoids the transit, except when the conjunction happens near one of the nodes.

The intervals between the transits are alternately a short one and a long one. The short ones are always of 8 years' duration, and the long ones alternately 121½ and 105½ years,

as the following table from Delambre's *Astronomy* will shew:

Dec. 6, 1631	June 7, 2004...121½ years.
Dec. 4, 1639... 8 years.	June 5, 2012... 8 ......
June 5, 1761...121½	Dec. 10, 2117...105½ ......
June 3, 1769... 8	Dec. 8, 2125... 8 ......
Dec. 8, 1874...105½	&c. &c.
Dec. 6, 1882... 8	

The list, continued according to this law, must be looked upon as giving the years when a transit may be expected; but an exact investigation must be made of the positions of the *Sun* and *Venus* before we can assert that a transit will or will not happen.

To account for the intervals which separate the transits:—We must know that the periodic times of Venus and of the earth round the sun are respectively 224·7 days and 365·256 days, so that the synodical period S, obtained from the equation

$$\frac{1}{S} = \frac{1}{224{\cdot}7} - \frac{1}{365{\cdot}256},$$

will consist of 584 days,

5 synodical revolutions will therefore require 2920· days,
8 revolutions of the earth........................ 2922· days;

therefore eight years after a transit, another conjunction will recur in the same part of the heavens very nearly, and where the planet will be again near its node.

We might thus almost expect to have a transit of Venus every eight years; but it is found that at the second of two such consecutive transits the latitude of Venus will differ from its former value by from 20′ to 24′. Another interval of eight years will bring a conjunction, at which the latitude of the planet will differ by from 40′ to 48′ from the first. This being greater than the sun's diameter, no third transit can take place; and we must wait until the conjunctions

occur near the other node, which, it appears, will require a period of $105\frac{1}{2}$ or $121\frac{1}{2}$ years.

Phases of the Planets.

368. Like the moon, the planets are opaque; and the light we receive from them is reflected sun-light. They will, therefore, present phases analogous to those of the moon.

It will be easily seen that, in the case of the inferior planets Mercury and Venus, the exterior angle of elongation SOV (fig. p. 234) may have all values from 0 to π; and, therefore, the phases will range through all values from a full round disc to a thin crescent, which itself vanishes at inferior conjunction. But, in the case of the superior planets, the angle which the radius of the earth's orbit subtends at the planet will always be very acute, and the supplementary angle, on the versine of which the phase depends (Art. 300), will always be very obtuse, so that nearly the whole of the disc will at all times be illuminated.

The inferior planets will present a half-disc when at their greatest elongation or angular distance from the sun, that is, when the line from the earth to the planet touches the planet's orbit. The greatest elongation of Venus varies from $45°$ to $47\frac{3}{4}°$, that of Mercury from $16°$ to $28\frac{1}{2}°$.*

369. *To find when Venus appears brightest.* The variations in the brightness of Venus are due, not only to the alteration in the phase of the planet, but also to the change in its distance from us. When approaching the earth, the diminution of illuminated area tends to diminish the brightness, but the distance is decreasing, and as the brightness varies inversely as the square of the distance, this latter cause will, at first, more than compensate for the loss

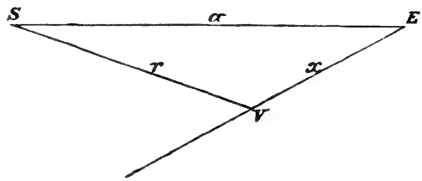
* These variations are due to the elliptic forms of the orbits. The excentricity of the orbit of Mercury is much greater than that of Venus.

due to the former, and Venus is found to be brightest when between its greatest elongation and inferior conjunction, at about 40° from the sun.

We shall investigate the position of greatest brightness on the supposition that the orbits both of the planet and of the earth are circular.

Let S, E, V be the sun, the earth, and Venus.

Let SE, or a, and SV, or r, be the radii of the two orbits.



EV, or x, the variable distance of Venus from the earth. Then

$$\text{brightness} \propto \frac{\text{area of phase}}{x^2} \propto \frac{1+\cos V}{x^2} \propto \frac{1+\dfrac{r^2+x^2-a^2}{2rx}}{x^2}$$

$$\propto \frac{1}{x^2} - \frac{a^2-r^2}{2rx^3} + \frac{1}{2rx}.$$

Therefore, equating to 0 the differential coefficient with respect to x,

$$-\frac{2}{x^3} + \frac{3(a^2-r^2)}{2rx^4} - \frac{1}{2rx^2} = 0,$$

$$x^2 + 4rx - 3(a^2 - r^2) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (A),$$

whence $$x = \sqrt{(3a^2 + r^2)} - 2r \ldots\ldots\ldots\ldots\ldots\ldots (B),$$

determines the distance of the planet from the earth when the brightness is a maximum.

Again, substituting in (A) for the sides the sines of the opposite angles to which they are proportional, we have

$$\sin^2(V+E) + 4\sin(V+E)\sin E = 3(\sin^2 V - \sin^2 E)$$
$$= 3\sin(V+E)\sin(V-E),$$

$$\sin(V+E) + 4\sin E - 3\sin(V-E) = 0,$$

$$-2\sin V + 4\cos V\tan E + 4\tan E = 0,$$

$$2\tan E = \frac{\sin V}{1+\cos V} = \tan\frac{V}{2} \ldots\ldots\ldots\ldots (C),$$

an equation given by Halley.

By means of this and $\frac{\sin V}{\sin E} = \frac{a}{r}$, we may deduce

$$\cos^2 E + \frac{4}{3}\frac{r}{a}\cos E = \frac{4}{3} \ldots\ldots\ldots\ldots\ldots\ldots (D),$$

which determines the corresponding elongation of the planet.

370. We shall not enter into a description of the different planets. They vary considerably in size, from Jupiter, whose equatorial diameter exceeds 90,000 miles, to the minor-planets, some of which are probably less than 100 miles in diameter. Some of the planets have several moons or satellites, others have none. One of them, Saturn, in addition to eight moons, is surrounded with several flat rings; but for a full account of these and other interesting details we shall refer to *The Solar System*, by J. R. Hind, Herschel's *Outlines of Astronomy*, Lardner's *Handbook of Astronomy*, &c.

APPENDIX.

THE CALENDAR.

371. WHAT we mean by the date of any event is the interval of time that separates that event from some definite epoch or era. For more than five centuries after the birth of Christ, the era in use was still, as before, the foundation of Rome. In the year 532 a Scythian monk, Dionysius Exiguus, proposed that all Christians should thenceforward take the Saviour's birth as the starting point for the expression of dates; and, from the result of his researches, he inferred that the birth took place on the 25th of December in the year 753 U.C.; but, as the beginning of the Roman civil year was on the 1st of January, *i.e.* only seven days later, it was found more convenient to make the Christian year begin on the same day. It was, therefore, decided that the year 1 of the Christian era should coincide with the year 754 of the foundation of Rome.

It must be remarked that there is no year 0 A.D. reckoned by chronologists—the year of the birth, that is, the year 753 U.C. being called by them the year 1 B.C.; the year preceding that, the year 2 B.C., etc.; so that to find the number of years between corresponding days of two years, the one B.C. and the other A.D., we must subtract 1 from the sum of the two.

We must refer to other works for the history of the various kinds of years which have, at different times and by different nations, been adopted, and we shall confine our attention to the Julian and the Gregorian calendars.

372. For the measure of intervals of time connected with civil life, there are two natural units—the solar day and the tropical year. The solar day as indicated by the successive returns of light and darkness, and the tropical year by the periodical changes of the seasons.

But it is obviously an essential condition of a unit that it shall be invariable, and the solar day is not so; and another necessary condition, when several units are employed, is that they shall be commensurable, which is not the case with the solar day and the tropical year.

The first difficulty is got over by taking for our unit the average of all solar days, or the mean solar day, as it is called (Art. 191), which answers the condition of perfect uniformity, and which, from the fact of mean noon being never very distant from apparent noon, is also in accordance with the recurrence of light and darkness. The mean day, therefore, with its subdivisions into hours, minutes and seconds, is the chronometric unit for short intervals of time.

The difficulty connected with the larger unit is of a different kind, and cannot be obviated in the same way. The tropical year is very nearly uniform,* but unfortunately it does not contain an exact number of days; and it would be an obvious inconvenience that a fraction of one day should belong to one year and the remaining portion of the same day to another year. Whatever length is adopted for the civil year, it must consist of an integral number of days, and must, moreover, retain a close connection with the tropical year, which contains 365·242216 days.

This can be managed by having civil years of two different lengths, the one less and the other greater than the tropical year; and the object of the calendar is to give definite rules

* It is found that the ecliptic changes its position in space under the influence of the planets. This change is very slow—about 48″ in a century—and produces a slow variation in the length of the tropical year, which is now about $4{\cdot}21^s$ shorter than in the time of Hipparchus.

for the order of succession of these two years of 365 and 366 days respectively, so that the connection may be preserved in the simplest manner.

373. Sosigenes, an Alexandrian astronomer, employed by Julius Cæsar to correct the confusion into which the calendar was perpetually falling, proposed the ingenious contrivance of bissextile or leap year. Three common years of 365 days were to be followed by a year of 366 days, thus giving to the average civil year a value of 365·25 days, which is a little more than the tropical year, the difference ·007784 days amounting to 1 day in about 128 years, or rather more than 3 days in 400 years. This important change came into operation in the 44th year B.C.

The next correction was made in 1582 A.D. by Pope Gregory XIII., with a view to take into account this difference of 3 days in 400 years; and, by this means, to avoid a change which was gradually bringing the festival of Easter more and more into the summer season, whereas the ecclesiastical regulations required that it should be celebrated just after the spring equinox.

The Gregorian calendar, which is now adopted by all Christians, except the Russians and the Greeks, is established on the following rule:—Three common years of 365 days are to be followed by a year of 366 days, as in the Julian calendar (the leap years being those whose number is divisible by 4 without remainder), except when the fourth year terminates a century, as 1700, 1800, &c., and then it becomes a common year; except, again, when the hundreds are divisible by 4, as 2000, 2400, &c., when it remains a leap year, as the Julian calendar would make it.

In 400 civil years, therefore, as determined by the Gregorian rule, there will be 97 leap years instead of 100, and the average length of the civil year will be $365\frac{97}{400}$ days or 365·2425 days. The tropical year contains 365·242216 days,

so the Gregorian rule makes the average civil year too long by 0·000284 days, producing an error of 1 day in about 4000 years.*

In the year 1582, when Pope Gregory made this reformation, he also omitted 10 nominal days of the month of October, the day after the 4th being called the 15th. This was done for the purpose of bringing back the vernal equinox to the 21st of March, which was the date of its occurrence in 325 A.D., when the Council of Nice was held and a rule was framed for the observance of the festival of Easter.

The *new style*, as it was called, was not adopted in England until the year 1752, when 11 days had to be omitted, and the month of September in that year contained only 19 days, which were numbered 1, 2, 14, 15, &c. In Russia the *old style* is still maintained, and the year 1800 has added another day to the difference of styles, so that the dates in Russia are now 12 days behind ours. Traces of the old style still linger with us in our 'Old Christmas-day,' 'Old Lady-day.'

Golden Number.

374. In connection with the calendar and its arrangements, we cannot pass over *Meton's lunar cycle*, which is the basis of the ecclesiastical rule for the determination of the moveable feasts—Easter, Whitsunday, &c.

In the earlier stages of astronomical science, when the festivals connected with the worship of the gods were dependent on the lunar phases, the difficulties attending the prediction of the days on which these festivals should be

* The Persian method of interpolation deserves to be mentioned. It was introduced in the 11th century, that is, five centuries before the Gregorian reformation, and is even more exact:—Three years of 365 days are followed by a year of 366 days, as in the Julian calendar; this is done seven times in succession, but the eighth period consists of five years—four common years followed by a year of 366 days. Thus in 33 years there are 8 leap years. Therefore, the average length of the Persian civil year is $365\frac{8}{33}$ days or 365·242424 days, which is in excess of the present length of the tropical year by ·000208 days, or 1 day in about 5000 years.

celebrated, were very great. About 432 B.C., Meton, an Athenian astronomer, discovered a relation between the length of the lunar month and the tropical year, which gave a simple rule for the solution of the problem. He found that after nineteen years the phases of the moon will recur on the same days of the same months; so that, without any calculation, it was only necessary to observe and to record the days of full moon during one of these cycles, in order to know them for all subsequent periods of nineteen years. These dates were ordered to be inscribed in letters of gold upon the public monuments, and in the modern use of the cycle of nineteen years, the number which marks the rank of any one of the years in the cycle, is still called the golden number of that year.

To shew how closely the Metonic cycle brings back the same phase of the moon to the same day of the year:—We know that the mean length of a lunation is 29·5306 days, and therefore 235 lunations occupy

$$235 \times 29{\cdot}5306 \text{ d.} = 6939{\cdot}69 \text{ days},$$

the length of the tropical year is 365·242 days, and 19 tropical years consist of

$$19 \times 365{\cdot}242 \text{ d.} = 6939{\cdot}60 \text{ days},$$

and the difference in nineteen years is only about 2 hours.

The first year of a cycle may be chosen arbitrarily, and it is found that the year 1 B.C. would begin the cycle now in use; therefore, to determine the golden number: *Add* 1 *to the date and divide by* 19, *the remainder is the golden number.* When the remainder is zero, the golden number is 19.

Thus, the golden number for 1874 is 13.

Moon's Age. Epact.

375. The moon's age at any time is the interval that has elapsed since the last new moon, that is, since conjunction (Art. 302).

The moon's age on the first of January is called the epact for that year. Now it happens, whether by accident or by design, that the first year of the cycle of golden numbers, that is, the year whose golden number is 1, has new moon on the first of January; therefore, the epact is 0 when the golden number is 1.

A tropical year consists of 365·242 days.

12 lunations occupy 29·5306 × 12 or 354·367 days.

Therefore a tropical year contains 12 lunations and 10·875 days.

At the beginning of the first tropical year of a golden cycle, the moon's age will be........................ 0·000 days.

At the beginning of the second it will be ..... 10·875 ......

At the beginning of the third .... 21·750 ......

At the beginning of the fourth.........32·625
or, subtracting one lunation ....29·531 } 3·094 ......

and so on.

If the civil year and the tropical year were of the same length and began together, we should have the following rule for finding the age of the moon at the beginning of each year:—Subtract 1 from the golden number, multiply by 10·875 and divide by 29·531, the remainder would be the moon's age. But the lunations are not always of the same length as we have here supposed; and, besides, the necessity for making the civil year consist of an exact number of days, and the consequent adjustment, by means of leap year, would interfere with this, and the following is the rule adopted: *Diminish the golden number by unity, multiply by* 11 *and divide by* 30. The remainder is the epact.

In some years of each cycle of 19, the difference between the epacts so found and the moon's age on the 1st of January may exceed one day; but the rule, besides being simple and sufficiently approximative, gives a definite law for the determination of the moveable feasts of the calendar, which is now the only important use of the epact.

THE TIDES.

376. The waters of the ocean, even in the calmest weather, and without any perceptible cause, rise and fall at nearly regular intervals; covering and uncovering broad expanses of sand along the coast, and otherwise changing the depth of the sea, in some instances by forty or fifty feet, and even more. This phenomenon is too striking and too important not to have from the earliest times attracted attention.

The rise and fall takes place twice every day, or, rather, twice in every lunar day, which occupies about $24^{h.}\ 50\frac{1}{2}^{m.}$; and the obvious connection between the *tides*, as these phenomena are called, and the motion of our satellite, led philosophers to attribute them to the action of the moon, long before the true nature of that action was known to them.

The explanation of the tides has been found in the theory of universal gravitation, which proves them to be necessary consequences of the joint attraction of the sun and moon combined with the rotation of the earth on its axis; and it may be shewn that the variations in the character of different tides are fully and satisfactorily accounted for by changes in the distances and relative positions of those luminaries. But the prediction of the circumstances of high and low water at any particular place becomes a problem of extreme difficulty, owing to local causes of disturbance which are unknown, or which we know not yet how to introduce into the investigation. Such are—obstacles to the fluid motion in the shape of headlands and islands, the varying depths of the bottom of the sea, the friction of the particles of water among themselves and against the shores, &c.

377. It is a strictly hydrodynamical problem, and as such was treated by Laplace, who, the first, attempted a general

solution on mathematical principles; but the difficulties attending this method of investigation are so great that he could not solve it in its generality, and he limited it by starting "with two suppositions which are inapplicable to the state of the earth. These are—that the earth is covered with water; and that the depth of this water is the same through the whole extent of any parallel of latitude." The Astronomer Royal, in his valuable paper on "Tides and Waves" in the *Encyclopedia Metropolitana*, has fully discussed Laplace's *Wave-theory*, and has further advanced the problem by considering the motion of the tidal wave in a canal, under a variety of different circumstances as to depth, friction, &c. This supposition cannot apply to all cases, and leaves the general theory still imperfect.

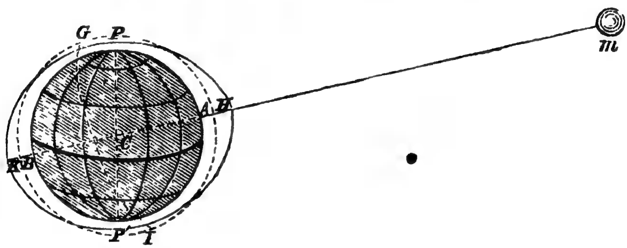
Newton, in the *Principia*, Lib. I., Prop. 66, Cor. 19, considered an equatorial belt of water forming a canal round the earth, and inferred that there would be *low water* under the moon. Sir G. Airy's investigation confirms this conclusion, but shews also that its truth depends on the depth of the canal; and that if the depth exceeded a certain amount (about 12 miles), there would be high water under the moon instead of low.* The problem of the tides is one of the most difficult in the whole range of mathematical subjects, and any further account of it would be foreign to, and beyond the scope of, the present work.

We shall, however, say a few words on what is known as the *equilibrium-theory* of Bernouilli, a theory confessedly inadequate and even based on erroneous principles (giving *high water* under the moon, irrespective of the depth); but yet, before Laplace's time, it was the only really successful attempt to determine the laws of tidal action and to reduce them to numerical computation.

* See an interesting paper by Mr. D. D. Heath in the *Phil. Mag.*, March, 1867

378. Bernouilli supposed the earth to have a spherical solid nucleus surrounded by a shell of water of uniform depth; then he demonstrated that if one of the bodies—say the moon—were *always vertically over the same point* of the surface, the waters would take the shape of a prolate spheroid pointing to the moon.

With this supposition let *AB* represent the solid earth surrounded by a shell *HK* of water, *AH* being the part just under the moon *m*, and *BK* the part most remote. The attraction of the moon, varying inversely as the square of the distance, will be greater on the parts of the earth



and on the waters about *A* than on the centre *C*, and this last greater again than on the parts about *B*. But the cohesion of the solid nucleus hinders any deformation, whereas the body of fluid may take a new form, and a disturbance of the water relatively to the earth will take place owing to the difference of attractions. The waters at *H* and over all the hemisphere *IAG* which is nearest to the moon, will be more attracted than the solid earth, and, flowing from the sides as far as the boundary *IG* towards *A*, will pile themselves up just under the moon and produce high water.

A similar effect will take place on the exactly opposite side of the earth at *B*, from the attraction on that half of the waters being less than on the earth itself, so that they are, as it were, left behind, and the waters flowing from *IG* towards *B* produce high water there at the same time as at *A*.

The heaping up of the waters on two opposite sides of the earth is due, then, not to the absolute attraction of the moon, but to the inequalities in the value of this attraction at different distances. There is, however, one point which deserves notice, because a misconception may easily arise; it is, that although the attraction on the waters differs from that on the earth by a greater quantity at A than at any other place, this attraction will not draw the water away from A. Its only effect will be to diminish in an imperceptible degree the gravitation of the water towards the earth; and if a ridge of land separated the waters at A from those which cover the rest of the earth, that is, if a small lake existed at A, the waters of this lake would not be drawn from the earth, and there would be no high tide.* The high water at A is due to the action of the moon on those parts of the hemisphere IAG, at a considerable distance from A, where the inequality of attraction mentioned above, although of less intensity than at A, acts in an oblique direction, and does not find itself entirely counteracted by gravity. Exerting a tangential action which has no opposing force, it causes the waters to flow towards A. The same remarks will apply to the high water at B.

But the moon does not remain constantly over the same place. The rotation of the earth about its axis PP', independently of the motion of the moon itself, changes the relative positions rapidly; and the theory, in order to adapt itself to the actual case, is obliged to make the supposition that the spheroidal form of the waters is *instantaneously* assumed in each new position of the earth. So that an observer situated on any meridian will have

* In the same manner, and for the same reason, the weight of a man is diminished as the moon approaches his zenith; and the effect of the sun being of the same kind, though of less intensity, we may say that at the time of new and full moon, when the two luminaries act together, the weight of a man is less at noon and midnight than at sunrise or sunset.

high water when his meridian comes into the position PAP', and again some twelve hours after when it comes into the position PBP', having in the interval passed through a position of low water. This explains the two tides observed in each lunar day. Another consequence also, which is evident from the figure, is that, except when the moon is in the equator, the apex of the waters will pass nearer to the observer, and the tide, therefore, will be higher at one of these transits than at the other.

As the earth rotates about its axis from west to east, the tide wave will obviously travel from east to west.

379. All that has been said about the effect of the attraction of the moon in raising the waters will equally apply to the sun, but the sun's action will be only five-elevenths of the moon's; the enormously greater mass of the sun being more than compensated by the increased distance.

There will then be two solar tides in each solar day, but the lunar tides being so much more powerful will override the solar ones; and the tides actually observed will be those due to the moon, hastened or retarded, increased or diminished, as the case may be, by combination with the solar ones.

Thus, at new-moon the sun and moon are in the same meridian, and at full-moon in opposite meridians; and, in either case, the two tidal waves coincide and a joint tide is produced, which is the sum of the two separate ones. These are called *spring tides*.

But in the quarters, the two bodies are 90° apart, and high water from the sun corresponds to low water from the moon, and *vice versâ*. The tide, therefore, has a diminished height, being the excess of the lunar over the solar tide. These are called *neap tides*.

From new-moon to the first quarter, and from full-moon

to the third quarter, the crest of the solar wave will be to the westward of the lunar wave, and the combination will produce a crest a little to the west, that is, in advance of the lunar wave, and high water will be hastened. In the same way during the second and fourth quarters, high water will be more or less retarded. This advance and retardation of the time of high water on the mean or average interval is called the *priming* and *lagging* of the tides.

380. Whether we suppose the crest of the tidal wave to be formed under the moon, or 90° from it, or in any other relative position, its advance will be modified by the interruption of the land, by the varying depth of the sea and by a variety of other causes which will delay the formation of the wave, and the crest will follow the moon at a greater or less interval. This interval is widely different at different places, and even in the same port it varies considerably, owing to the priming and lagging described above.

The interval between the instant of the moon's transit across the meridian on the day of new or full-moon, and the subsequent high water, is called the vulgar *establishment of the port.*

The name *corrected establishment* has been given by Dr. Whewell to the mean of all intervals taken every day, under all circumstances; and this mean is found not to be the same as the interval taken on the day of new or full-moon alone.

Cotidal-lines are curve lines which connect all those places which have high water at the same absolute instant of time; that is, all those places through which the crest or ridge of the tide-wave passes at the same instant.

381. In the motion of the tidal wave, we must remember that it is no bodily translation of the waters, but only an alteration of their form which produces the wave. There will, however, be currents caused by the tide, especially in

narrow channels; and it is found that these currents change their direction at different states of the tide, but not generally at the same time that the waters cease to rise or fall. Thus the current which flows forward at the time of high water continues to flow in the same direction for some two or three hours longer, and is called the *flood.* The return current, which accompanies low water and continues to flow back for two or three hours after, is called the *ebb.* The change from flood to ebb, or from ebb to flood, is called *slack water.*

382. We have said that the effect of the moon in producing tides is about $2\frac{1}{5}$ as great as the sun's. The following investigation will shew this, referring to the fig., p. 297:

Let the mass of the moon be represented by M, that of the earth by E, and let r be the radius of the earth, and $p \times r$ the distance of the moon from C. Then, g being the acceleration of gravity at the surface of the earth,

$$\text{attraction of moon on } C : g :: \frac{M}{p^2 r^2} : \frac{E}{r^2}.$$

$$\text{Therefore,} \quad \text{attraction of moon on } C = \frac{M}{E}\frac{g}{p^2},$$

$$\text{so,} \qquad \text{attraction of moon on } H = \frac{M}{E}\frac{g}{(p-1)^2};$$

$$\text{hence, disturbing force of moon on waters} = \frac{M}{E}\frac{2g}{p^3} \text{ approximately.}$$

Similarly, if S be the mass of the sun, and nr its distance from the centre of the earth,

$$\text{the disturbing force of sun on waters} = \frac{S}{E}\frac{2g}{n^3}.$$

Now $E = 80M$ and $S = 322000E$ (see note, p. 205),

$p = 60$ and $n = 23000$ (see Art. 262);

$$\text{therefore } \frac{\text{action of moon}}{\text{action of sun}} = \frac{(23000)^3}{80 \times 322000 \times (60)^3} = 2\cdot 2.$$

GREAT-CIRCLE AND MERCATOR'S CHARTS.

383. At sea, the problem the sailor has constantly presented to him is,—how to shape his course so as to arrive at his destination. For this he must be furnished with maps or charts on which the coast line of continents and islands is correctly traced, and all rocks and dangers properly indicated, together with the meridians and parallels. Then, day by day, he must determine the position of his ship, by finding his latitude and longitude according to the rules of Nautical Astronomy, some of which have been given in Chapters IX. and XXI. He must mark the place so found on the chart, and then determine the course or courses to be steered from this point to the port to which he is bound, or to those intermediate places where he must pass in order to avoid obstacles and dangers.

Of the various curves or lines by which he may go from one point to another on the surface of the sphere, there are two which deserve special attention. One is the shortest track, that is, the arc of a great circle passing through the two points; the other is the curve, called a *loxodrome*, which meets every meridian at the same angle.

In the great-circle track, the course or the angle which the great-circle makes with the various meridians is not constant, and the lengthy calculations necessary for finding these courses have always proved a drawback to its use among seamen; whereas the simplicity of keeping on one course, and the facility with which the loxodrome is drawn on the chart invented by Mercator, have caused the universal adoption of the method, notwithstanding the greater distance which it was well known the ships would have to go over.

Of late years, the great extension of our commerce, the introduction of steam vessels, and our increasing intercourse with Australia and other distant parts of the earth, have revived great-circle sailing, by the use of which the Australian journey can be shortened 1000 miles or more.

384. It is obvious that, since a spherical surface cannot be developed into a plane, a chart, however constructed, must give a distorted representation of the surface, some parts being unduly enlarged or contracted in proportion to others; and it is also obvious that the nature of the curve which represents the ship's intended track will depend on the character of the projection on which the chart is constructed.

If we suppose the eye at the centre of the sphere and the projection to be made by drawing lines through each point of the surface to meet a tangent plane, then every great-circle will be projected into a straight line, and the shortest route from point to point will, on the map, be the straight line which joins them. These are evidently the simplest kinds of charts for the purposes of great-circle sailing, but we must refer elsewhere for a full description of their construction and use.*

385. Mercator's chart is constructed on the principle that every loxodrome shall be represented by a straight line. Perhaps the simplest idea of the construction will be obtained as follows: Conceive a sphere representing the earth with all the meridians, parallels, coast-lines, &c. traced upon it, to be made of elastic material, and let a cylinder surround this sphere touching it along the equator. Now, imagine

* See a paper by the author in the *Transactions of the Cambridge Philosophical Society*, vol. X. part II., on "A chart and diagram for facilitating great-circle sailing." By means of this, the determination of the track, with its various courses and distances, is rendered as easy as Mercator's sailing.

the sphere to expand so as to fill up the cylinder, the expansion at each point ceasing as soon as that point comes into contact with the cylinder. Suppose also that the expansion is uniform in all directions; then, obviously, the relative proportions of all small parts will be maintained, but the scale will increase rapidly as we recede from the equator.

Let all the lines which were traced on the sphere now impress themselves on those parts of the cylinder with which they are in contact, and then let the cylinder be unrolled into a plane; this will be a Mercator's chart.

From the fact that each individual small element of the surface retains its *form*, we see that all angles will remain unaltered, and the meridians become parallel straight lines perpendicular to the equator, therefore a loxodrome, cutting all meridians at the same angle, will also become a straight line. So that the track of a ship which keeps on a constant course will be represented by a straight line, and this is the principle which renders Mercator's chart so useful to the navigator.

386. To find the distance from the equator to any parallel of latitude on a Mercator's chart.

Let a be the radius of the equator,

x ...... radius of the parallel in latitude ϕ,

s ...... length of the arc of the meridian between the equator and that parallel,

z ...... the corresponding distance on the chart,

Δs and Δz small increments of s and z.

By the method of expansion explained above, the circle, radius x, becomes a circle, radius a, therefore each small element of that parallel is increased in the ratio $x : a$. Hence Δs, which is changed to Δz, must ultimately be in the same ratio. Therefore

$$\frac{dz}{ds} = \frac{a}{x}.$$

The earth being a sphere, $x = a\cos\phi$, $s = a\phi$; therefore

$$\frac{dz}{d\phi} = a\sec\phi,$$

$$z = a\int_0^{\phi}\sec\phi d\phi = a\log\left\{\tan\left(45^\circ + \frac{\phi}{2}\right)\right\}.$$

387. If we take into account the spheroidal shape of the earth, and call e the excentricity of the ellipse, then, by Art. 253,

$$x = \frac{a\cos\phi}{(1-e^2\sin^2\phi)^{\frac{1}{2}}}, \quad y = \frac{a(1-e^2)\sin\phi}{(1-e^2\sin^2\phi)^{\frac{1}{2}}},$$

$$\left(\frac{ds}{d\phi}\right) = \sqrt{\left\{\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2\right\}} = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{\frac{3}{2}}}.$$

As before, $$\frac{dz}{ds} = \frac{a}{x},$$

$$\frac{dz}{d\phi} = \frac{a(1-e^2)}{\cos\phi\,(1-e^2\sin^2\phi)}$$

$$= \frac{a}{\cos\phi} - \frac{ae^2\cos\phi}{1-e^2\sin^2\phi},$$

whence $$z = a\log\tan\left(45^\circ + \frac{\phi}{2}\right) - \frac{ae}{2}\log\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)$$

$$= a\log\tan\left(45^\circ + \frac{\phi}{2}\right) - ae^2\sin\phi \text{ approximately.}$$

EXAMPLES AND PROBLEMS.

CHAPTERS I.—VI.

1. Shew that if α, β, be the angles subtended between a star and the horizon at two heights, a, b, in the same vertical line, the radius of the earth $= \left\{ \frac{\sqrt{(2b)} - \sqrt{(2a)}}{\beta - \alpha} \right\}$ very nearly.

2. Given the right ascensions and declinations of two stars, to find their distance.

3. The least angle which can be made with the horizon, by any great circle passing through the place of a star at a given time, is measured by the star's altitude.

4. The sun is at the same altitude at equal intervals of time before and after its passage over the meridian, supposing no change of declination to have taken place during the interval.

5. Find the declination of a star which, in a given latitude, rises in the north-east point.

6. Two declination circles PA, PB, make with each other a small angle ϵ at P, and from a point A (declination δ) in one of them an arc AB of a great circle is let fall perpendicularly on the other. Shew that the difference of declination of A and $B = \sin 2\delta \sin^2 \frac{1}{2}\epsilon$ nearly.

7. Given the distance of a planet from each of two stars, whose right ascensions and declinations are known, find the right ascension and declination of the planet.

8. If a ship be proceeding along a great circle, and the observed latitudes be λ_1, λ_2, λ_3, the distance traversed between the observations being in each case s, shew that

$$s = r \cos^{-1} \frac{\sin \lambda_1 + \sin \lambda_3}{2 \sin \lambda_2},$$

r denoting the earth's radius; and shew that the changes of longitude may also be found in terms of the three latitudes.

9. Given the latitudes and longitudes of two places where the inclination of the magnetic needle is nothing; to find the point of the terrestrial equator, which is cut by the magnetic equator, supposing it a great circle of the earth.

10. Two places in latitude 45° and whose difference of longitude is 90° are at two-thirds of the distance of places on the equator with the same difference of longitude.

11. If the same two stars rise together at two places, the places will have the same latitude. And if they rise together at one place and set together at the other, the places will have equal latitudes, but the one north and the other south.

12. Prove that all stars which rise at the same instant at a place within certain limits of latitude will, after a certain interval, be in a vertical great circle; and determine those limits.

13. Given the altitude of a known star when it is on the prime vertical, find the latitude of the place.

14. A known star rises in the north-east point, find from this circumstance the latitude of the place.

15. Two stars, whose right ascensions and declinations are known, were observed to rise at the same moment. Required the latitude of the place of observation.

16. It is observed that two stars whose right ascensions and declinations are known pass the prime vertical at the same instant. Required the latitude of the place of observation.

17. Two stars whose right ascensions and declinations are given have the same azimuth, and the altitude of one of them is known; the latitude of the place is required.

18. In a given latitude and longitude a vertical plane declines $a°$ from the south towards the west, find the place to whose horizon the plane is parallel.

19. Two known stars are seen at a given place A on the same vertical when at another place B they are rising together. Find the latitude and longitude of B.

20. Given the latitudes and longitudes of three places on the earth's surface, find the latitude and longitude of one equally distant from them all.

21. If C_1, C_2, C_3 be the lengths of the meridian shadows of three equal vertical rods on the same day at three different places on the same meridian, prove that the latitudes λ_1, λ_2, λ_3 of the places are connected by the equation

$$\frac{C_1(C_2-C_3)^2}{\tan(\lambda_2-\lambda_3)}+\frac{C_2(C_3-C_1)^2}{\tan(\lambda_3-\lambda_1)}+\frac{C_3(C_1-C_2)^2}{\tan(\lambda_1-\lambda_2)}=0.$$

22. How would an increase in the earth's velocity of rotation affect the latitude of a given place, supposing the form of the earth to remain unaltered?

23. How is it inferred that the axis of the earth's rotation always coincides sensibly with the same line of particles in the earth, and that it does not rapidly change its direction in space?

24. If the western pivot of a transit be α'' higher and β'' more to the north than the eastern, a star is unaffected whose north polar distance is

$$colat + \tan^{-1}\left\{\frac{\tan\alpha}{\sin\beta}\right\}.$$

25. If the head of the micrometer screw of the mural telescope be divided into sixty parts, and a whole turn of the screw carry it through 0·01 inches, what will be the correction of the reading when the screw-head indicates 15, and the focal length of the object-glass is 5 feet?

26. In obtaining the zenith distance of a star with the mural by reflection, the normal to the surface of the mercury at the point of reflection is slightly inclined to the vertical through the axis; shew that, on this account, the zenith-point, inferred from a double observation, is too great by the angle which the horizontal distance of the point from the axis subtends at the earth's centre, and the zenith-distance of the star deduced from the reflection observation is less than that from the direct observation by twice that angle.

CHAPTERS VII.—XIII.

27. When does the sun set at the point of the horizon opposite to that at which he rose?

28. Prove that, in the course of the year, the sun is as long above the horizon of any place as below it.

29. State some of the more striking differences in the phenomena presented by the sun on June 21st to observers in lat. 45° N. and 45° S. respectively.

30. Find the declination of the sun when, for a given place within the Arctic circle, the sun at mid-day just appears above the horizon.

31. If the equator were perpendicular to the ecliptic, describe the apparent diurnal motion of the sun throughout the year to an observer, firstly, at the pole, secondly, at the equator, and thirdly, in some intermediate position.

32. Which of the following great circles are fixed, and which of them would, if visible, appear fixed to an observer not aware of his own motion:—ecliptic, equator, meridian, solstitial colure?

33. Orion's belt being in the equator, and having about 5 h. 30 m. right ascension, during what part of the night will it be visible at the vernal and autumnal equinoxes?

34. Given the sun's meridian altitude and his midnight depression below the horizon, find the latitude of the place and the sun's declination.

35 At a certain place within the Arctic circle the sun did not set for two months; what was the latitude?

36. When the sun has a given north declination, shew at what parts of the earth he is visible, (1) during 24 hours, (2) during 12 hours.

37. At a place (lat. ϕ) in the Arctic circle, the sun will remain above the horizon at the summer solstice for $\frac{365\frac{1}{4}}{\pi}\cos^{-1}\{\cos\phi \operatorname{cosec}\omega\}$ days, neglecting the excentricity of the earth's orbit.

38. If λ be the latitude of a place at which, a month before the autumnal equinox, the day is as long as the longest day at a place in latitude λ', shew that $\tan\lambda = \tan\lambda' \sqrt{(1+3\sec^2\omega)}$ approximately, ω being the obliquity of the ecliptic.

39. Given the latitudes and longitudes of Cork and Rio Janeiro, shew how to find on what days the sun is on the horizon of both places at the same instant.

40. Shew that, neglecting the change of declination, the curve traced out by the extremity of the shadow of a vertical rod on a horizontal plane will be a conic section.

41. If the angle between the equator and the ecliptic were 15°, what fractional part of the earth's surface would be included in the torrid zones, the temperate zones, and the frigid zones respectively?

42. The true zenith distance of the polar star when it first passes the meridian is 46° 50′ 40″·75, and at the second passage 50° 25′ 50″·30. Required the latitude of the place.

43. The sun's meridian altitude was found on November 23rd to be 62° 41′ 15″. The chronometer indicated 7 h. 43 m. 14 s. The sun's declination at the preceding noon at Greenwich (as given by *Nautical Almanac*) was 20° 23′ 36″, and his hourly motion in declination 30″·7. From these data find the latitude of the ship.

44. At noon on March 25th, 1858, the sun's declination was 1° 42′ 29″, and the differences of right ascension between the sun and a star 13 h. 1 m. 49 s. At noon on September 18th, the sun's declination was 1° 59′ 43″, and it was distant from the star 1 h. 36 m. 0 s. in right ascension. At noon on September 19th, the declination of the sun was 1° 36′ 28″, and the difference of right ascension 1 h. 32 m. 24 s.; find the right ascension of the star, and that of the sun at the time of the first observation.

45. If the right ascension of a star be equal to its latitude, prove that its declination must be equal to its longitude

46. Indicate on a celestial sphere the points which have equal longitudes and Æ, and a latitude double their declination.

47. By observation it is found that the predicted place of a planet is erroneous in Æ and declination by the quantities α and β. Find from these the error in longitude and latitude.

48. *Assuming the sun's motion in longitude to be uniform*, shew that if α, β be the horary increments in right ascension and declination (expressed in circular measure), and α', β' the horary variations of α and β, then $\alpha' = 2\alpha\beta \tan\delta$, $\beta' = -\frac{1}{2}\alpha^2 \sin 2\delta$.

49. Trace the variations of the inclination of the ecliptic to the horizon of a given place, find its maximum and minimum values, and shew within what limits of latitude it can be a right angle.

50. Represent by a figure the position of the ecliptic relative to the meridian and horizon of Cambridge, at noon, on the autumnal equinox; under what circumstances, of place and time, would the ecliptic coincide with the prime vertical?

51. Assuming that the intensity of heat for different distances varies as the inverse square of the distance, shew that equal amounts of heat are received by the earth from the sun during the time of describing equal angles round it, in whatever part of the ellipse those angles may be situated.

52. Shew that the lengths of the four quarters of the year, beginning with the spring quarter, are approximately $Q + A\sin(B + \frac{1}{4}\pi)$ $Q - A\cos(B + \frac{1}{4}\pi)$, $Q - A\sin(B + \frac{1}{4}\pi)$, $Q + A\cos(B + \frac{1}{4}\pi)$; Q being one-fourth of the year, B the angular distance of the solar perigee from the autumnal equinox, and A a certain constant such that $A : 4Q :: e\sqrt{2} : \pi$, where e is the excentricity of the earth's orbit.

53. Shew that the time occupied by the sun in passing through the r^{th} sign of the zodiac, reckoning Aries the first, is approximately

$$M\left\{1 + \frac{24e \sin 15^\circ}{\pi} \cos(B + 15^\circ - r30^\circ)\right\}$$

where M is the 12^{th} part of the year, B the angular distance of the solar perigee from the autumnal equinox, and e the excentricity of the earth's orbit.

54. The times of the sun's rising and setting on November 1st are found from the tables to be 6 h. 56 m. and 4 h. 32 m. respectively; find approximately the equation of time.

55. Shew that the time of sunset is earliest some days before, and the time of sunrise latest some days after, the shortest day.

56. Assuming that the maximum amount of the equation of time due to the obliquity exceeds the maximum of that due to the excentricity, shew that the equation of time vanishes four times a year. How many times in the year would it vanish were the magnitudes of the two maxima reversed?

57. If the earth's perihelion coincided with a solstice, prove that, assuming the sun's true anomaly to be equal to $nt + 2e\sin nt$, and neglecting the square of the excentricity, the equation of time would vanish at the solstices, and when

$$e\cos\omega + \cos nt \sin^2\tfrac{1}{2}\omega = 0,$$

the true and mean suns being supposed to start with the same right ascension when the true sun is in a solstice.

58. When the sun's longitude is l, if x be the equation of time (expressed in angle) due to the obliquity of the ecliptic (ω) alone, shew that

$$\cot x = -\cot 2l - \cot^2 \tfrac{1}{2}\omega \operatorname{cosec} 2l.$$

59. Supposing the earth to describe a circle uniformly round the sun, the maximum value of the eqnation of time would be very nearly $\frac{1}{3}\pi n^2$ seconds, assuming the obliquity to be small and equal to n°.

60. At the head of the column for equinoctial time in March in the Nautical Almanac for 1848 was given "adding $\dfrac{0{\cdot}356624 \text{ d.}}{0{\cdot}114007 \text{ d.}}$" and for the days 21, 22, and 23, the days in the column are marked 364, 0, 1. Explain this clearly.

61. The mean time being 4 hours, find the corresponding sidereal time, having given the sun's mean daily motion $59'\ 8{\cdot}33''$, and the Æ at the preceding mean noon 144°.

62. If the sidereal time at mean noon were 16 h. 20 m. 48 s., what was the error of your watch at 2 o'clock, when a sidereal clock was at 18 h. 21 m., the sun's mean motion in longitude being $59'\ 8''{\cdot}33$ in a mean solar day?

63. The sun's apparent right ascension at mean noon Greenwich time, on June 1st, 1860, was 4 h. 38 m. 18·96 s. and on June 2nd, 4 h. 42 m. 24·73 s. Find the sun's apparent right ascension at 11 h. 20 m. A.M. on June 2nd, at a place 54° east.

And if the sun's right ascension at apparent noon on June 2nd be 4 h. 42 m. 24·34 s., find approximately the equation of time.

64. If the time be found by a single altitude, shew that a small error in the latitude will have no effect on the time when the body is in the prime vertical.

65. A ship leaves London at noon on a certain day, and arrives at New Orleans (90° W. long.) at noon, local time, on the thirtieth day afterwards. What is the actual time of passage?

66. The right ascensions of two stars which crossed the meridian of Greenwich at mean noon yesterday and to day respectively are $72^\circ\ 23'\ 21''{\cdot}15$ and $73^\circ\ 51'\ 29''{\cdot}55$; find the mean time at which a star whose right ascension is $317^\circ\ 21'\ 0''{\cdot}6$ crossed the same meridian yesterday.

67. If Jupiter revolves round the sun in 4320 of our days and round his axis in 10 hours, find by how much his mean solar day exceeds his sidereal day.

CHAPTER XIV.

68. Given the sun's altitude at 6 o'clock, and also when due east, find the latitude of the place.

69. Given the sun's declination, and that he is due east when half the time between his rising and 12 o'clock has elapsed, find the latitude of the place.

70. Find the latitude of the place at which the sun sets at 3 o'clock on the shortest day.

71. Two rods, the one 6 the other 8 feet high, are placed, on a given day, perpendicularly to the horizon, at a distance of 20 feet from each other. During the forenoon the extremity of the shadow of the first rod falls at the base of the second. In the afternoon the extremity of the shadow of the second falls at the base of the first. Required the latitude of the place and the azimuth of one rod seen from the other.

72. Given the point of the horizon at which the centre of the sun's disc rises, and the altitude of the point at which it crosses the meridian; find the time of the year and the latitude of the place.

73. Given the sun's altitude and azimuth, and the latitude of the place; to find the declination and the hour of the day.

74. In a given latitude determine the vertical in which the difference of the altitude of the sun in any two given days shall be a maximum.

75. The right ascension and declination of a star being given, and also the time of the year when it rises with the sun; find the latitude of the place.

76. Find the latitude of the place at which the sun sets at 10 o'clock on the longest day.

77. Given the latitude of the place and the sun's declination; to find his azimuth at 6 o'clock.

78. Given the latitude of the place and the sun's declination; find the time when the hour angle from noon and the sun's azimuth from the south are equal.

79. Given the sun's altitude and declination, and the sum of the azimuth and hour angle; to determine the latitude.

80. Given the latitude of the place and the sun's declination; find at what time of the day the azimuth of the sun increases slowest.

81. At a certain place in a given north latitude the sum of the sun's declination and altitude is known at a given hour in the morning; find the altitude and declination.

82. The length of the longest day at a given place is $14\frac{1}{2}$ hours. Find the latitude, supposing the obliquity of the ecliptic 23° 28′.

$$\log \tan 23^\circ\, 28' = 9{\cdot}63761,$$
$$\log \cos 71^\circ\, 15' = 9{\cdot}50710,$$
$$\log \tan 36^\circ\, 31' = 9{\cdot}86949.$$

83. The latitudes of two places on the earth's surface are complementary to each other, and on a given day the sun rises (n) hours earlier at one place than at the other; determine the latitude of each place.

84. A sphere of given radius is suspended in the air at a given place. Determine the figure of its shadow on a horizontal plane at a given day and hour; and shew that the length of the shadow varies as the secant of the sun's zenith distance.

85. Given the latitude of the place and the declination of the sun, the former being less than the latter, find at what time of the day the shadow of a vertical rod would have no angular motion.

86. Given the sun's diameter and the latitude of the place, determine the declination when the time the sun takes to rise is a minimum.

87. The direction of a street is θ° to the west of south; given that this street and the one at right angles to it are each exactly covered by the shadow of the block of houses between them, shew that if the breadth of the first street be a, the height of the houses h, and the altitude and azimuth of the sun ϕ and ψ respectively,

$$a \tan\phi - h \sin(\psi - \theta) = 0,$$

and find the breadth of the second street.

88. There are two walls of equal known height at right angles to each other, and running in known directions, shew how to find the sun's altitude and azimuth by observing the breadth of the shadows of the two walls at any given time. And prove that the sum of the squares of the breadths of the shadows will be the same whatever be the directions of the walls.

89. A person starts at sunrise to travel round the earth's equator. The sun crosses his meridian n times and he reaches the point of starting at sunset. Given the earth's radius, find his speed.

90. Prove that the perpendicular ascent of a star is always quickest in the prime vertical.

91. If the altitudes of a star be taken at the same place, on the same day, when it is in the same vertical on opposite sides of the meridian, shew that the sum of their tangents will be to the sum of their secants as the sine of the star's declination is to the sine of the latitude of the place.

92. Find the lowest latitude at which twilight lasts all night. Why is the mean duration of twilight shorter at the equator than elsewhere, and when is its duration the shortest?

93. Shew that the duration of twilight at the equator is

$$\frac{12}{\pi} \sin^{-1}(\sin 18^\circ \sec\delta) \text{ hours.}$$

94. Shew that the time at which the sun is south-east may be determined by means of the expression $\frac{1}{15}\{\theta - \sin^{-1}(\cos\theta\cos\phi\tan\delta)\}$, where δ is the sun's north declination, ϕ is the latitude of the place, and $\tan\theta = \sin\phi$.

95. The altitudes of a star when it crosses the meridian and the prime vertical of a place are a and a'; shew that if δ be the declination of the star, and ϕ the latitude of the place,

$$\cot\delta = \sec a \operatorname{cosec} a' - \tan a,$$

$$\cot\phi = \tan a - \sec a \sin a'.$$

96. If two stars rise simultaneously in azimuths which are supplementary, shew that one is as long above the horizon as the other is below it; and if $2z$ be the difference of their azimuths, and $2t$ hours the difference between their times of setting, shew that the latitude ϕ of the place is given by

$$\sin\phi = \cot z \tan(7\tfrac{1}{2}t^\circ).$$

97. To a spectator in the northern hemisphere, the sun, whose declination is 15° south, rises just two hours before noon; prove that the latitude of the place of observation

$$= \tan^{-1}\tfrac{1}{2}\sqrt{(3)}\sqrt{\left\{\frac{1+\frac{1}{2}\sqrt{(3)}}{1-\frac{1}{2}\sqrt{(3)}}\right\}}.$$

98. To a spectator on the deck of a vessel, sailing due west at the equator, at the rate of 10 miles an hour, the time occupied by the sun in rising on the day of the equinox is 2·154 m. Given that the angular breadth of the sun is 32′, find approximately the earth's radius.

99. Obtain the alteration in the length of the shortest day in latitude λ, corresponding to an alteration in the inclination of the earth's axis to the ecliptic.

100. At a place, the north latitude of which is 54°, and at a time of the year when the sun's north declination is 18°, shew that if h is the hour angle (ZPS) which the sun makes with the meridian at the moment of sunrise, then $\cos h = -\frac{1}{\sqrt{(5)}}$, and the sun rises shortly after 4 o'clock.

101. Give formulæ for determining how long the sun shines on a south wall, and how long on a west wall, in a given latitude on the shortest day.

102. Shew how to graduate a horizontal sun-dial, and find the limits beyond which it is useless to graduate it.

103. Find the length of the shadow of a man six feet high in latitude 60° at 8 A.M. on the 21st of March; find also the direction in which the shadow points.

104. In latitude 45°, at the equinox, find the time occupied by the sun in rising, assuming his diameter to be 30′.

105. Prove that in certain parts of the earth the ecliptic is perpendicular to the horizon every day, and find at what hour each day.

106. If h be the hour angle, and A the azimuth of a star at the instant when its altitude is equal to the latitude (ϕ) of the place of observation, shew that

$$\cos h = \tan\phi \tan(45 - \tfrac{1}{2}\delta),$$
$$\sin\tfrac{1}{2}A = \sec\phi \sin(45 - \tfrac{1}{2}\delta),$$

δ being the declination.

107. Shew that the velocity in azimuth at rising is the same for all stars at a given place.

108. At a place in latitude ϕ, a wall of height h has an azimuth of $a°$ to the east of south; shew that at the time of the equinox the wall casts no shadow at $\frac{1}{15}\tan^{-1}(\sin\phi\tan a)$ hours before noon, and at noon the breadth of the shadow is $h\tan\phi\sin a$.

CHAPTERS XV.—XVIII.

109. What would be the effect of refraction on a fixed star as seen by an observer on the moon's surface when the star is in the plane of the moon's orbit and passes behind the earth? The horizontal parallax being 60′, and the horizontal refraction 36′.

110. If r be the horizontal refraction, shew that the point of the compass where the sun rises is shifted by it,

$$\frac{\sin\phi}{\sqrt{\{\cos(\phi-\delta)\cos(\phi+\delta)\}}}\,r,$$

where ϕ is the latitude.

111. Calculate the alteration produced by refraction in the distance of two stars which has been taken by a micrometer.

112. It has been argued that since a degree of latitude increases in proceeding from the equator towards the poles, the whole circumference of the terrestrial meridian must be greater than 360 times a degree at the equator, that is, greater than the whole circumference at the equator, and that consequently the earth must be elongated instead of flattened at the poles. Shew the fallacy of this.

113. The moon's apparent declinations, as observed on the same day at stations in the southern and northern hemispheres respectively, were D and D'; the computed parallax corrections were p and p', and the increment in the moon's geocentric declination between the observations was d.

The previously determined equatorial horizontal parallax is slightly inaccurate; shew that the real value of

the constant : the value already obtained :: $D' - D - d : p + p'$.

114. The annual parallax of a double star is $0''\cdot307$, the apparent angular distance between its two parts $15''$, and the mean angular annual motion of revolution $40'$. Shew that the mass of the whole double star is nearly $\frac{2}{5}$ that of the sun.

115. Why is it not strictly true that the azimuth of a heavenly body is unaffected by parallax?

116. Assuming the distance of a body from the earth to be so great that the sine and circular measure of the parallax may be considered equal, shew that the locus of all bodies which, at a given instant and place, have their parallax in right ascension equal will be in a circular right cylinder touching the plane of the meridian along the axis of the earth.

117. What will be about the distance of a star of which the parallax is $2''$?

118. Find the right ascension of a star for which the aberration in right ascension vanishes at the summer solstice.

119. Find the coefficient of aberration in decimals of a second, when objects are viewed from a railway carriage travelling 60 miles an hour, the velocity of light being 200,000 miles a second.

If two planets describe circles in the same plane, when will the aberration in the position of one, as seen from the other, be greatest and least?

120. Find those stars for which, on a given day, the time of rising it not affected by aberration.

121. The apparent longitude of a star is recorded at various seasons of the year; how may it be ascertained whether the small changes, after eliminating any changes from other causes, are due to aberration or to parallax? and if due to both, what are the parts of the change separately due to the one and the other? Shew that the difference between apparent and mean longitude, arising from both causes combined, vanishes four times a year.

122. If $f(r, p) = 0$ be the equation to the earth's orbit, shew that $f\left(\frac{c^2}{p}, \frac{c^2}{r}\right) = 0$ will be the equation to the path which a star appears to describe in consequence of aberration.

123. Calculate the velocity of light when it is found that the difference between the declination of a star at the pole of the ecliptic in March and September is $40''$, the radius of the earth's orbit being 91 millions of miles, and the year consisting of 365 days.

124. If light moved but twice as fast as the earth in her orbit, in what part of the heavens would the pole star appear, at a place in north latitude 60°, at the time of the summer solstice?

125. At what season of the year is the aberration of a star in the position of the first point of ♈ greatest?

126. If the aberration of a star in longitude be the same as its aberration in latitude, prove that

$$\sin 2\lambda = 2 \cot(\odot - l),$$

where l, λ are the longitude and latitude respectively of the star, and $\odot$ the longitude of the sun.

127. A man runs a race starting with velocity v, the direction of the wind appears to him to make an angle a with his course. It then appears uniformly to veer round through an angle a during the race. Now the wind blows throughout uniformly, at right angles to his course. Shew that if τ be the time of the race, the length of the course is

$$v\tau \frac{\tan a}{a} \log(2 \cos a).$$

128. Shew that at any given time, all stars which lie in a certain great circle have no aberration in Æ.

129. Prove that all stars, of which the aberration in declination at any instant is zero, lie in the intersection of the celestial sphere with a cone, of which the circular sections are parallel respectively to the equator and to the great circle drawn through the sun's place perpendicular to the ecliptic.

130. What limit is there to the position of a place in order that, at some time in the day, a star in the ecliptic may have its error of aberration in a vertical plane?

131. Can a star be found whose real position is unaffected by parallax, refraction and aberration?

132. The star γ Draconis has rt. ascen. 17 h. 53 m. and *NPD* 38°·30′
„ 31 Camelopardali „ 5 h. 41 m. „ 30° · 9′
explain how Bradley, by observation of these stars, was able to separate the effects of aberration and nutation, and refer them to their real causes.

133. If a small change $\Delta\omega$ in the obliquity be due to nutation, shew that the corresponding changes in a star's right ascension and declination will be

$$\Delta Æ = -\tan\delta \cos Æ \,\Delta\omega,$$

$$\Delta\delta = \sin Æ \,\Delta\omega.$$

134. Explain why on account of precession the intervals between the passages of the meridian through the same star differ from a mean sidereal day.

If the colatitude of the star be less than that of the pole, this difference will vanish when the difference of longitudes of the pole and star is

$$\cos^{-1} \frac{\tan(\text{colat of star})}{\tan(\text{colat of pole})}.$$

135. What is the present longitude of a star which was the polar star in A.D. 66?

CHAP XIX.—APPENDIX.

136. At what seasons of the year will the crescent moon, when setting soon after the sun, have the line joining her horns most, and least, inclined to the horizon?

137. Shew that the full moon remains above the horizon of a place during nearly the whole of the night.

138. What must be the approximate age of the moon that she may be seen in the south at 7 o'clock in the morning? Will the convexity of the crescent appear to a spectator on his right hand or his left?

139. When the moon is a quarter old, what will be the earth's appearance to a spectator at the centre of her disc? At what points of his sky will the earth and sun respectively be seen? and what time of his day will it be?

140. How much should the moon's velocity of rotation about its axis be increased, in order that its whole surface might be seen in the course of an orbital revolution?

141. Find the least possible inclination to the horizon of the line joining the cusps of the moon.

142. The sidereal month being $27\frac{1}{3}$ days, find approximately the time at which the moon will rise to-morrow night if she rises to-night at 8 o'clock.

Find which side and, roughly, what proportion of her surface will be illuminated.

143. Find the length of the sidereal period of the moon, having given the synodic period and the length of the sidereal year.

144. If the moon's orbit be inclined at an angle ω to the equator, shew that when the moon rises at the same sidereal time, on two successive nights, the latitude is $90 - \omega$.

145. Prove that when the harvest-moon is most observable it will be accompanied by a central lunar eclipse.

146. In what parts of the earth may the rule that the moon's rising becomes later on successive evenings be reversed as regards the harvest full-moon?

147. If n, n' be the angular velocities of the moon about the earth, and of the earth about the sun, in orbits supposed circular, and if a be the moon's greatest elongation from the earth as it might be seen from the sun, shew that the time between the successive greatest elongations are alternately $\dfrac{\pi - 2a}{n - n'}$ and $\dfrac{\pi + 2a}{n - n'}$.

148. In an eclipse, does the obscuration begin on the eastern or western limb of the body eclipsed? In a solar eclipse, does the shadow of the moon move eastward or westward on the earth's surface?

149. If the moon's period were shortened to half its present

length, the other elements of her orbit remaining unchanged, what would be the result on the frequency and duration of eclipses?

150. Shew roughly that the duration of a total eclipse of the sun, for a place at the earth's equator, is doubled by the diurnal rotation of the earth.

151. If the inferior ecliptic limits are $\pm \epsilon$, and if the satellite revolves n times as fast as the sun, and its node regredes θ every revolution it makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node than the integer next less than $\frac{2(n-1)\epsilon}{n\theta+2\pi}$.

152. The angular diameters of the earth's shadow and of the moon, as seen from the earth's centre, being respectively 1° 30′ and 31′ nearly, and the apparent motion of the moon about the earth being 30′ per hour, find (1) the entire duration of a lunar eclipse, (2) the duration of the totality of the eclipse; the moon being in the node of her orbit when in opposition to the sun.

153. Find, roughly, the greatest latitude the moon can have at opposition, in order that an eclipse may take place; given the distances of the sun and moon 24000 and 60, their radii 100 and $\frac{1}{4}$, the earth's radius being 1.

Having also given the moon's period $27\frac{1}{3}$ days, find the greatest duration of an eclipse for the above values of the distances.

154. If θ be the circular measure of the inclination of the moon's relative orbit to the ecliptic, n^0 the angle between its line of nodes and the axis of the earth's shadow, μ^0, σ^0 the semi-diameters of the moon, and the section of the umbra; shew that, roughly, the duration of the eclipse $= 4\sqrt{\{(\sigma+\mu)^2 - n^2\theta^2\}}$ hours, and that $\left(\frac{\sigma+\mu-n\theta}{2\mu}\right)$ of the moon's diameter are eclipsed.

155. If Σ and Σ' be the semi-vertical angles of the cones of shadow and penumbra, θ the sun's apparent semi-diameter, then

$$2\sin\theta = \sin\Sigma + \sin\Sigma'.$$

156. The angular distance of Aldebaran from the moon's centre was observed at a certain place at 3 h. 40 m. to be 66° 14′; at Greenwich, at noon and at 3 h. the distances of the same objects were 65° 9′ 30″, and 66° 41′ 30″ respectively; determine the longitude of the place.

157. Supposing that the earth and a planet describe circles about the sun in the plane of the ecliptic, determine the geocentric motion of the planet. Also find when this motion is greatest and when it vanishes.

158. If a be the angle of elongation of an inferior planet when observed to be stationary from another planet, shew that $\cot a = \sqrt{(n^2+n)}$, when n is the ratio of the distance of the superior planet from the sun to that of the inferior; the orbits of the planets being supposed circular and in the same plane.

159. Supposing the paths to be circles in one plane, and the planet's distance to be 16 times that of the earth, find for how long its motion will be retrograde, having given $\cos^{-1}\frac{4}{13} = 72^\circ$ nearly.

160. Given that the periodic time of Venus is nearly two-thirds of that of the earth, find roughly what time of the year is indicated in these consecutive extracts from an almanac:

First month. Venus is an evening star this month. Enters Libra.

Next month. Venus is too close to the sun to be easily seen.

161. When a superior planet is stationary, shew that its angular distance from the sun is $\pi - \tan^{-1}\dfrac{n}{\sqrt{(1+n)}}$, where n is the ratio of the radius of its orbit to that of the earth.

162. If θ be the angle subtended at the earth by the sun and a stationary point of a planet's orbit, and ϕ be the greatest elongation of the planet, prove that

$$2\cot\theta = \sec\tfrac{1}{2}\phi + \operatorname{cosec}\tfrac{1}{2}\phi.$$

163. Supposing the orbits of two planets to be circular and to lie in the same plane, shew that the longitude of the interior, as seen from the exterior, will change more rapidly at the superior than at the inferior conjunction, but that its angular distance from the sun will change more rapidly at the inferior than at the superior conjunction.

164. A comet moving in a parabolic orbit, and a planet moving in a circular orbit, are in syzygy when the comet is in perihelion. Determine the ratio of the perihelion distance of the comet to the distance of the planet, in order that each may then appear stationary when seen from the other.

165. If the diameter of a planet's orbit, supposed circular, be to the diameter of the earth's orbit as $1 : \sqrt{(3)}$, find the synodic and sidereal periods.

Compare the phases of the earth and planet when the difference of their heliocentric longitudes is 30°. Also when it is 90°.

Shew that the brightness of the planet in the first case is to that in the second as 4 : 3.

166. The synodic period of Jupiter and the earth is 398 days. Find the periodic time of Jupiter.

167. Jupiter and Venus are evening stars, and stationary; find which way they will begin to move.

168. A ship has to sail from a place A to a place B, which is due east of A.

(a) Shew that the shortest course will not be due east.

(b) Find the direction in which the ship must sail to make the shortest course.

(c) Ex.: The latitude being 45° and the difference of longitude 90°.

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